Essays on the Interaction of Operational and Financial Decisions: Financing and Risk Management Perspectives

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Abstract

The dissertation comprises three chapters, each containing an essay on the interaction between operational and financial decisions of firms. The first chapter analyzes the interaction from an integrated risk management perspective. The second chapter focuses on the interaction from a financing point of view. The third chapter builds on the insights of the first two chapters and critically discusses the definitions of "operational hedging" that are proposed in the literature.

The first chapter of the dissertation focuses on the integrated risk management practices of the firms. Recent empirical findings make conflicting observations about the extent, interaction and effectiveness of operational and financial tools in risk management programs. Motivated by these observations, we analyze the integrated operational and financial risk management portfolio of a firm that determines whether to use flexible or dedicated technology and whether to undertake financial risk management or not. The risk management value of flexible technology is due to its risk pooling benefit under demand uncertainty. The financial risk management motivation comes from the existence of deadweight costs of external financing due to capital market imperfections. We characterize the optimal risk management portfolio as a function of firm size, technology and financial risk management costs, product market (demand variability and correlation) and capital market (external financing costs) characteristics. Our analysis contributes to the integrated risk management literature by characterizing the optimal risk management portfolio in terms of a more general set of operational and financial factors; providing the value and limitation of operational and financial risk management by explicitly modelling their costs and benefits; demonstrating the interactions between the two risk management strategies; and relating our theoretical results to empirical observations.

The second chapter of the dissertation focuses on the capacity investment decisions of the firms in imperfect capital markets. The Operations Management literature has traditionally implicitly assumed that capital markets are perfect in the sense that firms can raise sufficient capital to finance their operational investments. In this paper, we take a step in relaxing this perfect capital market assumption and formalize the capital market imperfections in the capacity investment setting. We focus on the interaction between a single firm that decides

on its technology choice (flexible vs dedicated), financial risk management level, capacity level, production quantities sequentially, and a single creditor that provides funds to the firm to finance its operational investments. The creditor has perfect information about the firm and offers two loan commitment contracts to the firm, one for each technology. The creditor incurs a fixed cost of bankruptcy if the firm defaults on the loan, and imposes an underwriting fee. The capital market imperfections, bankruptcy costs and underwriter fees, impose financing frictions on the firm. We derive the optimal technology, capacity, production, external borrowing and financial risk management decisions of the firm; and the creditor's optimal contracting decision in equilibrium. Our analysis contributes to the capacity investment literature by analyzing the effect of capital market imperfections on capacity investment and characterizing previously undocumented trade-offs that arise in imperfect capital markets and demonstrating that these trade-offs may change traditional insights concerning capacity investment derived under the perfect market assumption. For example, value of flexible technology may increase with increasing demand correlation and decreasing demand variability. Even with identical cost structures, dedicated technology may be preferred to flexible technology. These results are driven by the change in the equilibrium level of financing costs within imperfect capital markets.

The third chapter of the dissertation provides an extensive overview and synthesis of the existing literature on operational hedging. In particular, we focus on the treatment of operational hedging in the operations management literature under the light of the broader literature on the topic. Building on the insights of the first two chapters, we discuss several characteristics of the definitions of "operational hedging" proposed in the literature.

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Fontainebleau, January 2007

To my family,

Contents

Chapter 1

Introduction

This dissertation is about integrating the operational and financial decisions of the firm, understanding the value of such integration and discussing the possible interactions between these two sets of decisions from different perspectives. Two different financial decisions are considered, financing and financial risk management. In their seminal paper, Modigliani and Miller (1958) demonstrate that the firm's financing and investment decisions are independent within a perfect capital market. Within a same line of reasoning, it follows that financial risk management is irrelevant in a perfect capital market. In reality, capital market imperfections such as bankruptcy costs and underwriter fees do exist and interfere with the firm's operational decisions. Recently, the operations management (OM) literature and the finance literature have started to analyze the extent of this interaction by relaxing the perfect market assumption. Although the finance literature has already taken larger steps in this direction, most of the work in this stream considers the firm's operations either as a deterministic production function or at best as a random payoff distribution that is optimized over a single investment decision. Traditional OM literature provides a more realistic treatment of the firm's operations by going beyond the production function and recognizing the different levels of operational decisions. However, this literature generally (and often implicitly) assumes perfect capital markets. Absent this assumption, insights from traditional OM models may change. There is still a big gap in both literatures in our understanding of the interplay between the two sets of decisions in various environments. This dissertation takes a step in filling this gap. It comprises three chapters, each containing an essay on the interaction between operational and financial decisions of firms. The first chapter analyzes the interaction from an integrated risk management perspective. The second chapter focuses on the interaction mainly from a financing point of view. The third chapter builds on the insights of the first two chapters and provides a general conceptual framework to define operational hedging.

Summary of Chapter 2: The Interaction of Operational and Financial Decisions: An Integrated Risk Management Perspective

The first chapter of the dissertation focuses on integrated risk management practices of firms. Recent empirical findings make conflicting observations about the extent, interaction and effectiveness of operational and financial tools in risk management programs. Motivated by these observations, we analyze the integrated operational and financial risk management portfolio of a firm that determines whether to use flexible or dedicated technology and whether to undertake financial risk management or not. The risk management value of flexible technology is due to its risk pooling benefit under demand uncertainty. The financial risk management motivation comes from the existence of deadweight costs of external financing due to capital market imperfections. This chapter answers the following research questions:

1. What is the optimal risk management portfolio of the firm (defined as choosing flexible versus dedicated technology, and engaging in financial risk management or not) as a function of firm size, technology and financial risk management costs, product market conditions (demand variability and correlation) and capital market conditions (external financing costs)?

2. What are the fundamental drivers of the optimal risk management portfolio?

3. Are financial and operational risk management complements or substitutes?

4. What are the consequences of the interaction between financial and operational

risk management? What is the effect of financial risk management on operational decisions?

5. Can our results be used to support or refine existing empirical research?

Our main contributions are to model and analyze an integrated risk management problem that (i) yields structural results about the characteristics and drivers of an optimal risk management portfolio; (ii) provides managerial guidelines that can be used in designing risk management programs; and (iii) can be used to generate hypotheses that account for operational and product market characteristics to a greater extent than the existing empirical risk management literature.

We characterize the optimal risk management portfolio and find that three fundamental drivers explain the optimal portfolio choice: the robustness of the optimal capacity investment with respect to product market characteristics, the level of reliance on external financing and the opportunity cost of financial risk management. We show that firms can use financial risk management for speculative purposes with flexible technology, whereas they may prefer to hedge with dedicated technology. The reason is that firms with a limited internal budget can optimally increase their asset risk exposure to cover the higher fixed cost of flexible technology and invest in capacity to generate revenue. We demonstrate that engaging in financial risk management may induce the firm to change its technology decision; flexible technology and financial risk management can be complements or substitutes. This is a direct consequence of the difference between each technology regarding the counterbalancing value of financial risk management with respect to external financing costs.

We relate our theoretical findings to empirical observations concerning risk management practices of firms. Our results provide theoretical support for some observations and highlight additional trade-offs in others. For example, we establish that the value of financial risk management increases in external financing costs only for large firms and not for small firms. This is in contrast to existing understanding that this is true for any firm. We show that if firms use financial instruments only for hedging purposes, it is optimal for small firms to not undertake financial risk management;

existing arguments attribute this observation only to the fixed cost of establishing a financial risk management program. The distinction we make between large and small firms, and our results related to the effect of technology and product market characteristics on the risk management portfolio provide new hypotheses that can be tested empirically.

Summary of Chapter 3: Capacity Investment in Imperfect Capital Markets: The Interaction of Operational and Financial Decisions

The second chapter of the dissertation focuses on the interaction between operational and financial decisions in a capacity investment setting. The extant literature on capacity investment (often implicitly) assumes perfect capital markets (Van Mieghem 2003, p.294) and largely ignores the effect of financial decisions on the capacity investment decision. The objective of this chapter is to increase our understanding of how capital market imperfections affect technology choice and capacity investment.

To this end, we model a budget-constrained manufacturer who produces and sells two products. The firm chooses between flexible and dedicated technologies that incur variable investment costs, and determines the capacity level and the production quantities with the chosen technology. The firm's limited budget partially depends on a perfectly tradable asset. Thus, the firm is exposed both to product market (demand) and financial market (asset price) risk. The firm can relax its budget constraint by borrowing from a creditor. To capture capital market imperfections, we assume that the creditor incurs a fixed cost of bankruptcy if the firm defaults on the loan, and imposes an underwriting fee. The firm can use forwards written on the asset price to alter its budget distribution so as to counterbalance the effect of external financing costs arising from capital market imperfections.

Within this modelling setup, we answer the following research questions:

1. How do capital market imperfections affect capacity investment and operational performance?

2. For a given technology, what are the main drivers of capacity investment level

and operational performance in imperfect capital markets?

3. What are the main drivers of technology choice in imperfect capital markets?

4. Do these drivers differ from those in perfect capital markets and if so, what explains the difference?

5. What is the value of financial risk management in the creditor-firm interaction?

We demonstrate that an increase in capital market imperfection costs decreases the operational performance and the optimal capacity investment of the firm. This is because higher imperfection costs lead to higher financing costs in equilibrium. We show that the traditional insights on capacity investment that come from perfect markets may not continue to hold if there are imperfections in the capital markets. For example, value of flexible technology may increase with increasing demand correlation and decreasing demand variance. Even with identical cost structures, dedicated technology may be preferred to flexible technology. These results are driven by the change in the equilibrium level of financing costs within imperfect capital markets.

This chapter's major contribution is to the growing body of literature from OM and Finance fields that analyze the joint financing and operational decisions. The chapter's overall contribution is i) increasing the understanding of the effect of capital market imperfections on stochastic capacity investment; ii) demonstrating heretofore undocumented tradeoffs that arise in imperfect capital markets; iii) delineating the interaction between operational and financial decisions in capacity investment context. From a practical point of view, this chapter provides managerial insights about the effect of financial decisions on technology management. The results of this chapter are relevant for large firms that make large-scale investment decisions (e.g. semi-conductor firms) and start-up firms that are significantly financially constrained.

Summary of Chapter 4: Operational Hedging: A Review with Discussion
The fourth chapter of the dissertation provides an extensive overview and synthesis of the existing literature on operational hedging. Building on the insights of the

first two chapters, we discuss several characteristics of the definitions of "operational hedging" proposed in the literature. In the OM literature, operational hedging is equated to real options that provide different forms of operational flexibilities. These real options are defined to have hedging benefits by reducing the downside or the variance of the operating profits. We show that, under the light of broader literature, there are different operational tools other than the real options that are considered as operational hedges. We demonstrate that real options do not necessarily decrease the risk measure under consideration. We conclude with pointing out the necessity for a unifying operational hedging framework.

Chapter 2

The Interaction of Technology Choice and Financial Risk Management: An Integrated Risk Management Perspective

2.1 Introduction

This paper is about integrating operational and financial risk management and characterizing the drivers of the optimal integrated risk management portfolio. The two means of risk management are motivated by the existence of different market imperfection costs and utilize different tools. On the operational side, firms are exposed to demand and supply uncertainties in product markets. These uncertainties, which we call forms of *product market imperfection*, impose supply-demand mismatch costs. To manage these costs, firms rely on different types of operational flexibility that provide a better response to product market imperfections and counterbalance the effect of supply-demand mismatch costs. On the financial side, firms do not always have sufficient internal cash flows to finance their operations and depend on external capital markets to raise funds. The transaction costs in capital markets (bankruptcy costs, taxes, underwriter fees, agency costs etc.), which are forms of *capital market imperfection*, impose deadweight costs of external financing on firms. To manage these costs, firms rely on different types of financial instruments written on tradable assets with which their cash flows are correlated. These financial instruments engineer the internal cash flows of firms to meet their optimal investment needs and counterbalance the effect of external financing costs.

Despite responding to two different types of market imperfection, operational and financial risk management interact with each other: The choice of operational risk management has implications for financial risk management and vice versa. Therefore, operational and financial risk management should be viewed as constituting an integrated risk management portfolio. In practice, most corporate-level risk management programs of non-financial firms focus only on financial risk management (Bodnar et al. 1998). At the same time, a number of large non-financial firms are becoming more interested in operational solutions to manage their risk exposures (Business Week 1998). Due to the existence of both product and capital market imperfections in practice, using both risk management tools – and doing so in an integrated fashion – is important.

The academic literature on risk management has largely documented the value and effectiveness of each risk management tool in isolation. Relatively little progress has been made in understanding their interactions and the main drivers of an optimal integrated risk management portfolio. The objective of this paper is to enhance our understanding of integrated risk management. Our main contributions are to model and analyze an integrated risk management problem that (i) yields structural results about the characteristics and drivers of an optimal risk management portfolio; (ii) provides managerial guidelines that can be used in designing risk management programs; and (iii) can be used to generate hypotheses that account for operational and product market characteristics to a greater extent than the existing empirical risk management literature.

To this end, we model a budget-constrained manufacturer who produces and sells two products. Product demands are random, which is the *product market imperfec-*

tion, and correlated. The firm chooses between flexible and dedicated technologies that incur fixed and variable costs, and determines the capacity level of the chosen technology. Because of its risk pooling benefit, the flexible technology is the firm's *operational risk management tool.* The firm's limited budget partially depends on a perfectly tradable asset. The firm can relax its budget constraint by borrowing from external markets, but borrowing incurs external financing costs that originate from *capital market imperfections.* Forwards written on the asset price can be used as the firm's *financial risk management tool* to alter the budget distribution and help counterbalance the effect of external financing costs. The fixed and variable investment costs of flexible technology are higher than those of dedicated technology, and financial risk management has a fixed cost. Therefore, it may be undesirable to use these tools despite their value. In this rich but parsimonious model, we answer the following research questions:

1. What is the optimal risk management portfolio of the firm (defined as choosing flexible versus dedicated technology, and engaging in financial risk management or not) as a function of firm size, technology and financial risk management costs, product market conditions (demand variability and correlation) and capital market conditions (external financing costs)?
2. What are the fundamental drivers of the optimal risk management portfolio?
3. Are financial and operational risk management complements or substitutes?
4. What are the consequences of the interaction between financial and operational risk management? What is the effect of financial risk management on operational decisions?
5. Can our results be used to support or refine existing empirical research?

We derive the optimal integrated risk management portfolio and the related capacity, production, financial risk management and external borrowing levels, the majority of them in closed form. Our analysis reveals that there are three fundamental drivers that explain the optimal portfolio choice: the robustness of the optimal capacity investment level to product market conditions, the level of reliance on external financing and the opportunity cost of financial risk management. These drivers work in opposite directions for large and small firms due to differences in their borrowing needs under financial risk management. As a result, the size of the firm is highly relevant – the same underlying conditions lead to different optimal portfolio choices as a function of firm size. Conversely, it may be optimal for small and large firms to choose the same optimal portfolio for entirely different reasons. These results generate managerial insights and guidelines for designing an integrated risk management program.

Our analysis clearly illustrates the intertwined nature of operational and financial risk management strategies. We show that firms can use financial risk management for speculative purposes with flexible technology, whereas they may prefer to hedge with dedicated technology. The reason is that firms with a limited internal budget can optimally increase their asset risk exposure to cover the higher fixed cost of flexible technology and invest in capacity to generate revenue. We demonstrate that engaging in financial risk management may induce the firm to change its technology decision; flexible technology and financial risk management can be complements or substitutes. This is a direct consequence of the difference between each technology regarding the counterbalancing value of financial risk management with respect to external financing costs.

We relate our theoretical findings to empirical observations concerning risk management practices of firms. Our results provide theoretical support for some observations and highlight additional trade-offs in others. For example, we establish that the value of financial risk management increases in external financing costs only for large firms and not for small firms. This is in contrast to existing understanding that this is true for any firm. We show that if firms use financial instruments only for hedging purposes, it is optimal for small firms to not undertake financial risk management; existing arguments attribute this observation only to the fixed cost of establishing a financial risk management program. The distinction we make between large and small firms, and our results related to the effect of technology and product market characteristics on the risk management portfolio provide new hypotheses that can be tested empirically.

We note that all of the results[1] obtained are analytical and are valid for any demand and asset price distribution with positive and bounded support. With these results, we contribute to the growing operations management literature that incorporates financial considerations in operational decision making. In the next section, we provide more detail about how our work contributes to the existing literature. In §3.3, we describe the model and discuss the basis for our assumptions. §2.4 analyzes the optimal strategy of the firm, culminating in a characterization of the optimal risk management portfolio. §2.5 and §2.6 flesh out the results of the previous section to describe the impact of various factors on the optimal portfolio choice. We analyze the value and effect of integrated decision making by comparing with the non-integrated benchmark in §2.7. In §2.8, we discuss the robustness of our results to our assumptions. §3.9 concludes.

2.2 Literature Review

In this section, we review the streams of literature related to our paper and delineate our contributions to each stream. The operations management literature has documented the risk management value of operational flexibility. Starting with the influential studies of Huchzermeier and Cohen (1996), Cohen and Huchzermeier (1999) and Kouvelis (1999), this stream delineates the value of various operational flexibilities (e.g. technology flexibility, geographical diversification, postponement) in the firm's network structure, referred as operational hedges, in managing demand-side product market imperfections (Van Mieghem 2003, 2006, Aytekin and Birge 2004, Kazaz et al. 2005). We refer the reader to Boyabatlı and Toktay (2004) for a recent review of papers in this stream. A number of papers take this analysis further and study the interaction between different operational flexibilities of firms (Bish and Wang 2004, Goyal and Netessine 2005, Chod et al. 2006a, Dong et al. 2006). This stream of papers (often implicitly) assumes perfect capital markets and hence there are neither deadweight costs of external financing nor any value for financial risk management. We demonstrate the effect of external financing costs and financial risk management on the value of operational risk management, and document several interactions between operational and financial risk management.

The finance literature on risk management, in turn, focuses on financial risk management (e.g. forwards, options, etc.) and typically does not consider product market imperfections and operational risk management. The majority of this literature i) provides different explanations for the existence of financial risk management that are based on different types of capital market imperfections; or ii) focuses on the optimal use of financial instruments in a variety of settings. Since the focus of these papers is financial risk management, the interactions between the two risk management strategies are not studied. We refer the reader to Fite and Pfleiderer (1995) for a review of the first stream and Brown and Toft (2001) for a review of the second.

There are a few theoretical papers that study the firm's integrated risk management portfolio choice. In operations, Chod et al. (2006b) and Ding et al. (2005) analyze the interaction between financial risk management and different types of operational flexibility, where financial risk management is motivated by the risk aversion of the decision maker. Chod et al. (2006b) analyze whether financial risk management complements or substitutes operational flexibility. They demonstrate that this depends on whether the optimal flexibility level increases or decreases with financial hedging. We show that financial and operational risk management can again be either complements or substitutes under external financing, but the driver is firm size. Ding et al. (2005) is closest to our paper in terms of its research objective. They study the integrated operational (postponement) and financial risk management (currency options) decisions of a multinational firm and delineate the value of each risk management strategy under demand and exchange rate uncertainty. In a numerical study, they show that engaging in financial risk management alters the robustness of operational decision variables (capacity) with respect to demand variability and changes the strategic decision variables (global supply chain structure). We demonstrate similar results analytically. In addition, we analyze the effect of external financing costs, demand correlation and firm size on the optimal risk management portfolio. Incorporating the costs of each risk management strategy enables us to also explore the limits of their use.

In finance, Mello et al. (1995) and Chowdry and Howe (1999) model a multinational firm that has sourcing flexibility (sourcing from both domestic and foreign production facilities is possible) and that uses financial instruments to manage the exchange rate risk. These papers demonstrate the value of sourcing flexibility in conjunction with financial risk management. The focus of these papers is mainly financial risk management, and they do not consider a detailed representation of the firm's operations. Our analysis generates a number of insights about integrated risk management in a more detailed model of firm operations.

All of these papers assume that financial risk management is costless, in which case financial risk management is trivially included in the optimal risk management portfolio since it has positive value. In contrast, the fixed cost of financial risk management (e.g. software and personnel costs) can be a deterrent in practice. Motivated by this observation, we incorporate a positive fixed cost for engaging in financial risk management. This makes whether to engage in financial risk management or not a nontrivial question. The answer to this question goes beyond a boundary invest/do not invest decision divorced of the other decision variables: Under a budget limit and external financing costs, the effective cost of financial risk management is larger than its fixed cost because the firm may need to borrow an additional amount as a result of incurring this fixed cost. Therefore, engaging in financial risk management has an impact on the level of other decisions variables. Similarly, the fixed cost of the technology investment has a subtle effect on the optimal portfolio. These interactions add interesting dimensions to the optimal risk management portfolio.

In contrast to the theoretical finance research, the empirical finance literature has paid more attention to operational risk management, as reviewed in Smithson and Simkins (2005). This literature either statistically or qualitatively attributes a number of empirical observations to the firm's operational risk management capabilities, which we discuss these observations in detail in §2.5 and §2.6. We contribute to this stream in a number of ways: We provide theoretical support for some empirical observations and delineate additional trade-offs in some others; we provide alternative explanations to some observations that are based on the interplay between the two risk management strategies; and we identify potential future empirical research avenues.

In summary, our major contribution is to the integrated risk management literature. We contribute to this literature by i) characterizing the optimal risk management portfolio in terms of a more general set of operational and financial factors; ii) providing the value and limitation of each risk management strategy by explicitly modelling the costs and benefits of each strategy; iii) demonstrating the interactions between the two risk management strategies; and iv) relating our theoretical predictions to empirical observations.

Note that we have made a distinction between papers that augment the financial risk management analysis with operational risk management versus operational decisions only. Up to this point, we focused on the former, which involves a type of flexibility that can be used for risk management (and subsumes a number of operational decisions). The latter focuses only on operational decisions in analyzing financial risk management.

In the latter stream, we highlight Froot et al. (1993) from the finance literature since their modelling of the financial risk management motive is the same as in our paper. The authors use a concave increasing investment cost function to capture the operational dimension. They demonstrate that financial risk management adds value by generating sufficient internal funds to finance operational investments when there exist deadweight costs of external financing. We extend their framework by formalizing the operational investments (by incorporating product market characteristics, and technology and production decisions), and by imposing a cost for financial risk management. We illustrate that some of their predictions continue to hold, whereas some change due to the interplay between financial and operational decisions.

In the operations literature, Birge (2000), Chen et al. (2004), Gaur and Seshadri (2005), and Caldentey and Haugh (2005, 2006) document the value of financial risk management when the operating cash flows are correlated with a financial index. The financial risk management rationale is the risk-aversion of the decision maker in the latter three papers. Among these papers, we can link our paper to Caldentey and Haugh (2005) who motivate financial risk management by imposing a budget constraint on the firm, but without the possibility of external financing. This can be viewed as a special case of our model: When the external financing cost is sufficiently high, the firm never borrows. The external borrowing feature of our model is an important determinant of the risk management portfolio: the reliance on external borrowing determines the technology choice and the value of financial risk management with each technology.

Finally, our work is related to two other streams in operations management. The stochastic capacity investment literature analyzes the question of flexible versus dedicated technology choice with demand-side (uncertain demand) and supply-side (unreliable supply) product market imperfections. We refer readers to Van Mieghem (2003) for an excellent review and to Tomlin and Wang (2005) for a specific focus on the supply-side imperfection. As highlighted in Van Mieghem (2003), stochastic capacity models (often implicitly) assume perfect capital markets. We demonstrate that under financing frictions, there exist additional trade-offs in technology choice: the level of reliance on external financing and the value of financial risk management with each technology.

A second stream relaxes the perfect capital market assumption and models the firm's joint financial and operational decisions (Lederer and Singhal 1994, Buzacott and Zhang 2004, Babich and Sobel 2004, Xu and Birge 2004 and Babich et al. 2006). The primary focus of these papers is to analyze the effect of external financing costs and the financing decision on operational decisions. They demonstrate the value of integrated financing and operational decision making. We extend the interaction argument in these papers by considering another facet of financial decisions, financial risk management. Our analysis reveals that the effect of external financing costs are largely dependent on the value of financial risk management and that technology choice is a key determinant of the firm's reliance on external markets: the higher investment cost of flexible technology requires higher external financing levels than dedicated technology.

2.3 Model Description and Assumptions

We consider a monopolist firm selling two products in a single selling season under demand uncertainty. The firm chooses the technology (dedicated versus flexible), the capacity investment level and the production level so as to maximize expected shareholder wealth. Differing from the majority of traditional stochastic technology and capacity investment problems, we model the firm as being budget constrained, where the budget partially depends on a hedgeable market risk. We allow the firm to undertake financial risk management to hedge this market risk, and to borrow from external markets to augment its budget. After operating profits are realized, the firm pays back its debt; default occurs if it is unable to do so.

We model the firm's decisions as a three-stage stochastic recourse problem under financial market and demand risk. In stage 0, the firm chooses its integrated risk management portfolio. The firm decides its technology choice (flexible or dedicated), whether to engage in financial risk management, and if so, its financial risk management level under demand and financial market risk. In stage 1, the financial market risk is resolved and the financial risk management contract (if any) is exercised; these two factors determine the internal cash level of the firm. The firm then determines the level of external borrowing and makes its capacity investment using its total budget (internal cash and borrowed funds). In stage 2, demand uncertainty is resolved and the firm chooses the production quantities for each product. Subsequently, the firm either pays back its debt or defaults. In the remainder of this section, we define the firm's objective and discuss the assumptions concerning each decision epoch in detail. We discuss the robustness of our results with respect to the majority of these assumptions in §2.8.

Assumption 1 *The firm maximizes the expected (stage 2) shareholder wealth by maximizing the expected value of equity. The shareholders are assumed to be risk-*

neutral and the risk-free rate r_f is normalized to 0. Shareholders have limited liability.

The main goal of corporations is to maximize shareholder wealth. The expected shareholder wealth is a function of the expected cash flows to equity of the firm and the required rate of return of the shareholders. By assuming the risk neutrality of shareholders, we focus on maximizing the expected equity value of the firm. The required rate of return is the risk-free rate, which is normalized to 0 by assumption. Although the shareholders are risk-neutral, the existence of external financing costs creates an aversion to the downside volatility of the internal cash level in stage 1: The firm may be forced to underinvest in capacity at low internal cash level realizations because of external financing costs. This creates a motivation for undertaking firm-level financial risk management activities (Froot et al. 1993).

2.3.1 Stage 0

In this stage, the firm determines its technology choice $T \in \{D, F\}$, whether to use financial risk management, and if so, the financial risk management level H_T under financial market and demand uncertainty. The flexible technology (F) has a single resource that is capable of producing two products. The dedicated technology (D) consists of two resources that can each produce a single product.

Assumption 2 *Technology T has fixed (F_T) and variable (c_T) capacity investment costs. The fixed cost of the flexible technology is higher than that of the dedicated technology; $F_F \geq F_D$. The variable capacity investment cost of the two dedicated resources are identical. Both technologies are sold immediately at the end of the selling season at a reduced price of $\gamma_T F_T$ where γ_T is the salvage rate and $0 \leq \gamma_T < 1$. The firm commits to technology in this stage whose fixed cost is incurred in stage 1.*

Since flexible technology is generally more sophisticated than dedicated technology, the fixed cost of flexible technology is assumed to be higher. The stage 0 commitment of the firm to technology choice can be justified by the lead time of the acquisition (if outsourced) or the development time (if built in-house) of the technology. When the technology is resold, because of depreciation and liquidation costs, the fixed cost of the technology cannot be fully retrieved ($\gamma_T < 1$).

Assumption 3 *The firm uses a loan commitment contract to finance its capacity investment and to cover the fixed cost of the committed technology. The terms of the contract are known at stage 0, while borrowing takes place at stage 1.*

Loan commitment is a promise to lend up to a pre-specified amount at pre-specified terms. In practice, most short-term industrial and commercial loans in the US are made under loan commitment contracts (Melnik and Plaut 1986). At stage 0, the firm owns the right to a loan contract that can be exercised in stage 1. We discuss the characteristics of the loan commitment contract in Assumption 15 of stage 1.

Assumption 4 *At stage 0, the firm has rights to a known internal stage 1 endowment* (ω_0, ω_1). *Here,* ω_0 *represents the cash holdings and* ω_1 *represents the asset holdings of the firm. The asset is a perfectly tradeable asset that has a known stage 0 price of* α_0 *and random stage 1 price of* α_1. *The random variable* α_1 *has a continuous distribution with positive support and bounded expectation* $\overline{\alpha}_1$.

With this assumption, in stage 0, the firm knows that the value of its endowment will be $\omega_0 + \alpha_1\omega_1$ in stage 1, where α_1 is random; this is the financial market risk in our model. This representation is consistent with practice: In general, firms hold both cash and tradable assets on their balance sheet, such as a multinational firm that has pre-determined contractual fixed payments denominated in both domestic and foreign currency, or a gold producer that produces a certain level of gold that is exposed to gold price risk. In these examples, the asset price α_1 represents the exchange rate and the gold price in stage 1, respectively. Although the cash and the asset holdings are certain, the price of the asset makes the stage 1 value of the internal endowment random. The firm can use financial risk management tools to alter the distribution of this quantity.

Assumption 5 *The firm uses forward contracts written on asset price* α_1 *to financially manage the market risk. There is a fixed cost* F_{FRM} *of engaging in financial* *risk management that is incurred in stage 0 by transferring the rights of the firm's claims ω_0 and ω_1, in proportions β and $1-\beta$. Forward contracts are fairly priced. We restrict the number of forward contracts H_T such that the firm does not default on its financial transaction in stage 1.*

Forward contracts are the most prevalent type of financial derivatives used by non-financial firms (Bodnar et al. 1995). The fixed cost of financial risk management (F_{FRM}) includes the costs of hiring risk management professionals, and purchasing hardware and software for risk management; it is independent of the number of forward contracts used. In a recent survey, non-financial firms report this fixed cost as the second most important reason for not implementing a financial risk management program (Bodnar et al. 1998). Since we focus on loan commitment contracts and the firm can borrow from external markets only at stage 1, F_{FRM} is deducted in stage 0 from the firm's stage 1 endowment by transferring the rights of the claims ω_0 and ω_1 with β and $1-\beta$ proportions respectively. In other words, rights for βF_{FRM} of the cash holdings and $\frac{(1-\beta)F_{FRM}}{\alpha_0}$ of the asset holdings are transferred in stage 0. This leaves the firm with a stage 1 endowment of $(\omega_0^{FRM}, \omega_1^{FRM}) \doteq (\omega_0 - \beta F_{FRM}, \omega_1 - \frac{1-\beta}{\alpha_0} F_{FRM})$. The firm can only engage in financial risk management if these quantities are non-negative, or equivalently, if $F_{FRM} \leq \min\left(\frac{\omega_0}{\beta}, \frac{\alpha_0 \omega_1}{1-\beta}\right)$. Since the firm is exposed to external financing costs in stage 1, there is an opportunity cost associated with F_{FRM}: The firm has lower internal cash in stage 1 and may need to borrow more from external markets after paying for F_{FRM}. The fair-pricing assumption ensures that the firm can only affect the distribution of its budget in stage 1 – and not its expected value – by financial risk management. We restrict the feasible set of forwards to the range $\left[-\frac{\omega_0^{FRM}}{\bar{\alpha}_1}, \omega_1^{FRM}\right]$. Within this range of forwards the firm never defaults on its financial transaction in stage 1. This ensures that we can use default-free prices in forward transactions.

2.3.2 Stage 1

In stage 1, the market risk α_1 is resolved. The value of the firm's internal endowment and the exercise of the financial contract (if any) determine the firm's budget B. In this stage, the firm can raise external capital if the budget is not sufficient to finance the desired capacity investment. The firm determines the amount of external borrowing and the capacity investment level under demand uncertainty.

Assumption 6 *With the loan commitment contract, the firm can borrow up to credit limit E from a unit interest rate of $a > r_f = 0$. The face value of the debt $e_T(1+a)$ is repaid out of the firm's assets in stage 2. The firm has physical assets of value P (e.g. real estate) that are pledged to the creditor as collateral. The loan is secured (fully collateralized), i.e. $E(1+a) \leq P$. The physical assets are illiquid; they can only be liquidated with a lead time. The value of the physical assets P is sufficient to finance the budget-unconstrained optimal capacity investment level of the firm. The salvage value of technology ($\gamma_T F_T$) cannot be seized by the creditor among the firm's assets. Any possible costs that may be incurred in the borrowing process by the creditor (e.g. fixed bankruptcy costs) are charged ex-ante to the firm in a.*

We assume that the loan commitment is fully collateralized by the firm's physical assets P, i.e. $E(1+a) \leq P$, since most bank loans are secured by the company's assets (Weidner 1999) and modelled as such (Mello and Parsons 2000). Although the loan is fully collateralized, if the firm's final cash position is not sufficient to cover the face value of the debt, the firm cannot immediately liquidate the collateral assets to repay its debt since the physical assets are illiquid. Under limited shareholder liability, this leads to default, in which case the creditor can seize these physical assets, liquidate them and use their liquidation value to recover the loan. The salvage value of technology is assumed to be non-seizable; the creditor cannot use the salvage value to recover the face value of the loan. We also assume that the creditor's transaction costs associated with default (e.g. fixed bankruptcy costs) are charged to the firm ex-ante in the unit borrowing cost.

A positive unit financing cost ($a > 0$) and a credit limit less than the value of the collateralized asset ($E < P$) can be interpreted as the deadweight costs of external financing that arise from capital market imperfections: If the capital markets are perfect (i.e. there are no transaction costs, default related costs, information asymmetries), then the contract parameters are determined such that the loan is fairly valued in terms of its underlying default exposure. Since we focus on a collateralized loan, in the absence of default-related deadweight costs, there is no risk for the creditor associated with default. Consequently, in perfect capital markets, the fair unit financing cost of the loan commitment contract would be the risk-free rate ($a = 0$), and the credit limit would be the value of collateralized physical asset ($E = P$). If there are capital market imperfections, then $a > 0$ and $E < P$ would be obtained in a creditor-firm interaction. Therefore, although we assume that they are exogenous parameters in this paper, a positive unit financing cost ($a > 0$) and a credit limit less than the value of the collateralized asset ($E < P$) can be interpreted as capturing the deadweight costs of external financing that arise from capital market imperfections. This parallels the assumptions in Froot et al. (1993) who take the external financing costs as exogenous and state that they can be argued to arise from deadweight costs associated with capital market imperfections.

In a creditor-lender equilibrium, the (endogenous) contract parameters need not be identical for each technology. In §2.8, we discuss conditions under which our results with identical contract parameters are valid in a general equilibrium setting, and refer the reader to Boyabatlı and Toktay (2006b) for an analysis of equilibrium contract (a_T^*, E_T^*) for each technology in a creditor-firm Stackelberg game.

To conclude, we note that our external financing cost structure provides a parsimonious model that is consistent with real-life practices; allows us to implicitly capture capital market imperfections and enables us to preserve tractability.

2.3.3 Stage 2

In this stage, demand uncertainty is resolved. The firm then chooses the production quantities (equivalently, prices) to satisfy demand optimally. If the firm is able to repay its debt from its final cash position, it does so and terminates by liquidating its physical assets. Otherwise, default occurs. In this case, because of the limited liability of the shareholders, the firm goes to bankruptcy. The cash on hand and the ownership of the collateralized physical assets are transferred to the creditor. The firm receives the remaining cash after the creditor covers the face value of the debt from the seized assets of the firm.

Assumption 7 *Price-dependent demand for each product is represented by the iso-elastic inverse-demand function* $p(q_i; \xi_1) = \xi_i q_i^{1/b}$ *for* $i = 1, 2$. *Here,* $b \in (-\infty, -1)$ *is the constant elasticity of demand, and* p *and* q *denote price and quantity, respectively.* ξ_i *represents the idiosyncratic risk component.* (ξ_1, ξ_2) *are correlated random variables with continuous distributions that have positive support and bounded expectation* $(\overline{\xi}_1, \overline{\xi}_2)$ *with covariance matrix* Σ, *where* $\Sigma_{ii} = \sigma_i^2$ *and* $\Sigma_{ij} = \rho\sigma_1\sigma_2$ *for* $i \neq j$ *and* ρ *denotes the correlation coefficient.* (ξ_1, ξ_2) *and* α_1 *have independent distributions. The marginal production costs of each product at stage 2 are 0.*

2.4 Analysis of the Firm's Optimal Risk Management Portfolio

In this section, we describe the optimal solution for the firm's technology choice, and the levels of financial risk management, external borrowing, capacity investment and production. A realization of the random variable s is denoted by $\tilde{s}$ and its expectation is denoted by $\overline{s}$. Bold face letters represent vectors of the required size. Vectors are column vectors and $'$ denotes the transpose operator. Vector exponents are taken componentwise. $\mathbf{xy}$ denotes the componentwise product of vectors $\mathbf{x}$ and $\mathbf{y}$ with identical dimensions. We use the following vectors throughout the text: $\boldsymbol{\xi} = (\xi_1, \xi_2)$ (product market demand), $\mathbf{K_F} = K_F$ (flexible capacity investment) and $\mathbf{K_D'} = (K_D^1, K_D^2)$ (dedicated capacity investment). Pr denotes probability, $\mathbf{E}$ denotes the expectation operator, $\chi(.)$ denotes the indicator function with $\chi(\varpi) = 1$ if ϖ is true, $(x)^+ \doteq \max(x, 0)$ and $\Omega^{01} \doteq \Omega^0 \bigcup \Omega^1$. Monotonic relations (increasing, decreasing) are used in the weak sense otherwise stated. Table 3.1 summarizes the decision variables. Table 5.1 that summarizes other notation and all proofs are provided in Appendix I (Chapter 5). We solve the problem by using backward induction starting from stage 2.

Stage	Name	Meaning
Stage 0	$T \in \{D, F\}$	Technology choice, dedicated or flexible
	H_T	Number of forwards with technology T
Stage 1	e_T	Borrowing level with technology T
	$\mathbf{K_T}$	Capacity investment level with technology T
Stage 2	$\mathbf{Q_T}$	Production quantity with technology T

Table 2.1: Decision variables by stage

2.4.1 Stage 2: Production Decision

In this stage, the firm observes the demand realization $\tilde{\boldsymbol{\xi}}$ and determines the production quantities $\mathbf{Q_T}' = (q_T^1, q_T^2)$ within the existing capacity limits to maximize the stage 2 equity value.

Proposition 1 *The optimal production quantity vector in stage 2 with technology $T \in \{D, F\}$ for given $\mathbf{K_T}$ and $\tilde{\boldsymbol{\xi}}$ is given by*

$$\mathbf{Q_D^*} = \mathbf{K_D}, \quad \mathbf{Q_F^*} = \frac{K_F}{\tilde{\xi}_1^{-b} + \tilde{\xi}_2^{-b}} \, \tilde{\boldsymbol{\xi}}^{-b}.$$

Since the unit production cost is zero, the firm optimally utilizes the entire available capacity. With dedicated technology, the optimal individual production quantities are equal to the available capacity levels for each product. With flexible technology, the firm allocates the available capacity K_F between each product in such a way that the marginal profits for each product are equal.

2.4.2 Stage 1: Capacity Choice and External Financing

In this stage, the firm exercises the forward contract H_T (if the firm has already decided to engage in financial risk management at stage 0) and observes the asset price $\tilde{\alpha}_1$. With fair pricing, the strike price of the forward is equal to $\overline{\alpha}_1$. The stage 1 budgets with and without financial risk management are therefore $B_{FRM}(\tilde{\alpha}_1, H_T) \doteq \omega_0^{FRM} + \tilde{\alpha}_1(\omega_1^{FRM} - H_T) + \overline{\alpha}_1 H_T$ and $B_{-FRM}(\tilde{\alpha}_1) \doteq \omega_0 + \tilde{\alpha}_1\omega_1$, respectively. We henceforth suppress $\tilde{\alpha}_1$ and H_T and denote the available budget realization by $\tilde{B} \in [0, \infty)$. For given $\tilde{B}$ and T, the firm determines the optimal capacity investment level $\mathbf{K}_\mathbf{T}^*(\tilde{B})$ and the optimal external borrowing level $e_T^*(\tilde{B})$.

Proposition 2 *The optimal capacity investment vector $\mathbf{K}_\mathbf{T}^*(\tilde{B})$ and the optimal external borrowing level $e_T^*(\tilde{B})$ for technology $T \in \{D, F\}$ with a given budget level $\tilde{B}$ are*

$$\mathbf{K}_\mathbf{T}^*(\tilde{B}) = \begin{cases} \mathbf{K}_\mathbf{T}^\mathbf{0} & \text{if } \tilde{B} \in \Omega_T^0 \doteq \{\tilde{B} : \tilde{B} \geq c_T \mathbf{1}'\mathbf{K}_\mathbf{T}^\mathbf{0} + F_T\} \\ \overline{\mathbf{K}}_\mathbf{T} & \text{if } \tilde{B} \in \Omega_T^1 \doteq \{\tilde{B} : c_T \mathbf{1}'\mathbf{K}_\mathbf{T}^\mathbf{1} + F_T \leq \tilde{B} < c_T \mathbf{1}'\mathbf{K}_\mathbf{T}^\mathbf{0} + F_T\} \\ \mathbf{K}_\mathbf{T}^\mathbf{1} & \text{if } \tilde{B} \in \Omega_T^2 \doteq \{\tilde{B} : \tilde{B} \geq \widehat{B_T},\ c_T \mathbf{1}'\mathbf{K}_\mathbf{T}^\mathbf{1} + F_T - E_T \leq \tilde{B} < c_T \mathbf{1}'\mathbf{K}_\mathbf{T}^\mathbf{1} + F_T\} \\ \overline{\overline{\mathbf{K}}}_\mathbf{T} & \text{if } \tilde{B} \in \Omega_T^3 \doteq \{\tilde{B} : \widehat{B_T} \leq \tilde{B} < c_T \mathbf{1}'\mathbf{K}_\mathbf{T}^\mathbf{1} + F_T - E_T\} \\ \mathbf{0} & \text{if } \tilde{B} \in \Omega_T^4 \doteq \{\tilde{B} : 0 \leq \tilde{B} < \widehat{B_T}\} \end{cases} \quad (2.1)$$

$$e_T^*(\tilde{B}) = \left(c_T \mathbf{1}'\mathbf{K}_\mathbf{T}^*(\tilde{B}) + F_T - \tilde{B}\right)^+ \chi\left(\tilde{B} > \widehat{B}_T\right). \quad (2.2)$$

Here, $\chi(.)$ is the indicator function and $\widehat{B_T}$ is the unique budget threshold for technology $T \in \{F, D\}$ such that the firm optimally does not borrow ($e_T^*(\tilde{B}) = 0$) and does not invest in capacity ($\mathbf{K}_\mathbf{T}^*(\tilde{B}) = \mathbf{0}$) for $\tilde{B} \leq \widehat{B_T}$.

The explicit expressions for the capacity vectors in the proposition are given in (5.23) in the proof. $\mathbf{K}_\mathbf{T}^\mathbf{0}$ is the optimal capacity investment in the absence of a budget constraint (the "budget-unconstrained optimal capacity"). If the budget realization is high enough to cover the corresponding cost $F_T + c_T \mathbf{1}'\mathbf{K}_\mathbf{T}^\mathbf{0}$ ($\tilde{B} \in \Omega_T^0$), then $\mathbf{K}_\mathbf{T}^*(\tilde{B}) = \mathbf{K}_\mathbf{T}^\mathbf{0}$ with no borrowing. Otherwise, for each budget level $\tilde{B} \in \Omega_T^{1234}$, the firm determines to borrow or not by comparing the marginal revenue from investing in an additional unit of capacity over its available budget with the marginal cost of that investment including the external financing cost, $(1 + a)c_T$. For $\tilde{B} \in \Omega_T^1$, the budget is insufficient to cover $\mathbf{K}_\mathbf{T}^\mathbf{0}$, and the marginal revenue of capacity is lower than its marginal cost. Therefore, the firm optimally does not borrow, and only purchases the capacity level $\overline{\mathbf{K}}_{\mathbf{T}}$ that fully utilizes its budget $\tilde{B}$. For $\tilde{B} \in \Omega_T^{23}$, the marginal revenue of capacity is higher than its marginal cost $(1+a)c_T$. Therefore, the firm optimally borrows from external markets to invest in capacity. $\mathbf{K}_{\mathbf{T}}^{\mathbf{1}}$ is the optimal capacity investment with borrowing, in the absence of a credit limit (the "credit-unconstrained optimal capacity"). If the budget realization and the credit limit can jointly cover its cost, $\mathbf{K}_{\mathbf{T}}^{\mathbf{1}}$ is the optimal capacity investment; otherwise, the firm purchases the capacity level $\overline{\overline{\mathbf{K}}}_{\mathbf{T}}$ that fully utilizes its budget and its credit limit. For $\tilde{B} \in \Omega_T^4$, the firm must borrow to be able to invest in technology, but the total cost of the capacity that can be purchased with the remaining $\tilde{B} + e_T - F_T$ cannot be covered by the expected revenue it generates for any e_T. Therefore, the firm optimally does not borrow and does not invest in capacity. Appendix 5 in Chapter 5 characterizes $\widehat{B}_T$ and provides a closed-form expression for a subset of parameter values.

The optimal external borrowing level $e_T^*(\tilde{B})$ is such that the firm borrows exactly what it needs to cover its capacity investment. Since production is costless, the firm does not incur any further costs beyond this stage. Moreover, since the face value of the debt is always deducted from the firm's assets, the firm cannot transfer wealth from the creditor to shareholders by borrowing more money than what is needed for its capacity investment. Therefore, the firm only borrows for funding the capacity investment, which yields (3.2).

The optimal expected (stage 1) equity value of the firm with a given budget level $\tilde{B}$, $\pi_T(\tilde{B})$, is obtained in closed form (Equation 5.29 in Appendix I (Chapter ??)).

Corollary 1 *$\pi_T(\tilde{B})$ strictly increases in $\tilde{B}$ for $\tilde{B} \geq 0$, and is concave in $\tilde{B}$ on $[\widehat{B_T}, \infty)$. It is not concave in $\tilde{B}$ on $[0, \infty)$.*

As we will see in 3.4.3, this structure has implications for the optimal financial risk management level.

2.4.3 Stage 0: Financial Risk Management Level and Technology Choice

In this stage, the firm decides on the technology choice $T \in \{D, F\}$, whether to engage in financial risk management (FRM) and if so, the financial risk management level H_T, the number of forward contracts written on the stage 1 asset price α_1. The optimal expected (stage 0) equity value $\Pi^*(\mathbf{W})$ as a function of the internal (stage 1) endowment $\mathbf{W}' = (\omega_0, \omega_1)$ is

$$\Pi^*(\mathbf{W}) = \max\left\{\Lambda^{-FRM}, \Lambda^{FRM}, \omega_0 + \overline{\alpha}_1\omega_1 + P\right\}. \tag{2.3}$$

Here, Λ_{FRM} and Λ_{-FRM} denote the expected (stage 0) equity value of the better technology with and without financial risk management (FRM), respectively, where Λ_{FRM} is calculated at the optimal risk management level H_T^*. In (3.3), the firm compares these equity values with $\omega_0 + \overline{\alpha}_1\omega_1 + P$, the expected (stage 0) equity value of not investing in any technology. §3.4.3 derives H_T^*, §3.4.3 characterizes the optimal technology choice with and without FRM, and §2.4.3 characterizes the solution to (3.3). This characterization is valid for any continuous α_1 and $\boldsymbol{\xi}$ distribution with positive support and bounded expectation.

Financial Risk Management

The expected direct gain from the financial contract is 0 due to the fair pricing assumption. At the same time, financial risk management affects the distribution of the stage 1 budget $B_{FRM}(\alpha_1, H_T)$, which is used to finance the firm's capacity investment after paying for the fixed cost commitment. In choosing H_T, the goal of the firm is to engineer its budget to maximize the expected gain from the technology commitment made in stage 0. When $H_T > 0$ ($H_T < 0$), the firm decreases (increases) its exposure to the asset price risk α_1. Following Hull (2000, p.12), we refer to the first case as *financial hedging*, and to the second as *financial speculation*. We call $H_T = \omega_1^{FRM}$ *full hedging* because it isolates the budget from the underlying risk exposure. We call $H_T = -\frac{\omega_0^{FRM}}{\overline{\alpha}_1}$ *full speculation* because it maximizes the firm's asset risk exposure within the feasible range of forward contracts. Proposition 3 characterizes H_T^*.

Proposition 3 *There exists a unique technology fixed cost threshold $\overline{F}_T$ such that*

(i) If $F_T \leq \overline{F}_T$, then the firm fully hedges ($H_T^* = \omega_1^{FRM}$).

(ii) If $F_T > \overline{F}_T$ then

1. *if $\omega_0^{FRM} + \overline{\alpha}_1 \omega_1^{FRM} \leq \widehat{B}_T$, then full speculation is optimal ($H_T^* = -\frac{\omega_0^{FRM}}{\overline{\alpha}_1}$);*
2. *if $\omega_0^{FRM} + \overline{\alpha}_1 \omega_1^{FRM} > \widehat{B}_T$, $H_T^* \in \left\{ \left\{ H_T < \frac{\widehat{B}_T - \omega_0^{FRM}}{\overline{\alpha}_1} \right\} \cup \left\{ \omega_1^{FRM} \right\} \right\}$ and is distribution dependent.*

The structure of π_T is key to these results. If π_T is a concave function of the available budget $\tilde{B}$ on $[0, \infty)$, then full hedging is optimal. This follows by Jensen's inequality: For concave π_T, $\mathbf{E}\left[\pi_T(B_{FRM}(\alpha_1, H_T))\right] \leq \pi_T(E[B_{FRM}(\alpha_1, H_T]) = \pi_T(\omega_0^{FRM} + \overline{\alpha}_1 \omega_1^{FRM})$, the equity value under full hedging. However, π_T is not concave if $\Omega_T^4 \neq \emptyset$, i.e. if there is a budget range in which the firm would not invest in capacity in stage 1 despite having made the technology investment in stage 0. This happens when the fixed cost of the technology investment is too high to leave sufficient funds for a profitable capacity investment.

Below the fixed cost threshold $\overline{F}_T$, $\Omega_T^4 = \emptyset$, π_T is concave, and full hedging is optimal. Above this threshold, H_T^* depends on the expected value of the internal (stage 1) endowment $\omega_0^{FRM} + \overline{\alpha}\omega_1^{FRM}$, which is also the budget available to the firm under full hedging. When this value is lower than $\widehat{B_T}$, the firm would optimally not invest in capacity if it were to fully hedge. Instead, the firm optimally chooses to increase its exposure as much as possible so as to maximize the probability of realizing high-budget states in which it is able to invest in capacity and generate revenue from its technology investment. (This also increases the probability of realizing low-budget states, but the outcome in those states does not change - no capacity investment is optimal.) For $\omega_0^{FRM} + \overline{\alpha}_1 \omega_1^{FRM} > \widehat{B}_T$, the optimal risk management level is distribution dependent and a full characterization is not possible without making further assumptions.

Technology Choice

We now turn to the technology selection problem with and without financial risk management. The choice T^* between flexible versus dedicated technology is determined by a unit cost threshold that makes the firms indifferent between the two technologies.

Proposition 4 *For given technology cost parameters* (F_T, γ_T) *and financing cost scheme* (a, E), *and under the financial risk management level* H_T^* *for each technology, there exists a unique variable cost threshold* $\bar{c}_F(c_D, \mathbf{H}^*)$ *such that when* $c_F < \bar{c}_F(c_D, \mathbf{H}^*)$ *it is more profitable to invest in flexible technology* $(T^* = F)$. *Without financial risk management, there is a parallel threshold* $\bar{c}_F(c_D, \mathbf{0})$. *These thresholds increase in* c_D, F_D, γ_F *and demand variability* (σ), *and they decrease in* F_F, γ_D *and the demand correlation* $(\rho)^2$. *With symmetric fixed costs and salvage rates,*

$$\bar{c}_F(c_D, \mathbf{H}^*) = \bar{c}_F(c_D, \mathbf{0}) = \bar{c}_F^S(c_D) = c_D \left(\frac{\mathbf{E}^{-b}\left[\left(\xi_1^{-b} + \xi_2^{-b} \right)^{-\frac{1}{b}} \right]}{\mathbf{E}^{-b}[\xi_1] + \mathbf{E}^{-b}[\xi_2]} \right)^{-\frac{1}{b+1}} \geq c_D, \quad (2.4)$$

where the equality only holds if the product markets are deterministic $(\sigma = 0)$, *or the product markets are perfectly positively correlated* $(\rho = 1)$ *and* $\boldsymbol{\xi}$ *has a proportional bivariate distribution.*

The comparative statics results developed here are used in §2.6 to analyze the drivers of the firm's optimal risk management portfolio. The threshold $\bar{c}_F^S(c_D)$ is independent of unit financing cost a, credit limit E, and engaging in financial risk management. Although these factors do have an effect on the equity value of each technology, the differential value of this effect is never sufficient to induce the firm to alter its technology decision. This threshold is independent of α_1 and valid for any distribution of $\boldsymbol{\xi}$. The threshold $\bar{c}_F^S(c_D)$ is a variant of the mix flexibility threshold in Chod et al. (2006a), and has the same structure. It is interesting to note that the same threshold structure is valid despite the existence of external financing costs and financial risk management policy in the symmetric cost case.

Due to the risk pooling benefit of flexible technology, we have $\bar{c}_F^S(c_D) \geq c_D$. Proposition 4 shows that there is no risk pooling benefit ($\bar{c}_F^S(c_D) = c_D$) only if the product market demand is deterministic, or the multiplicative demand uncertainty is perfectly positively correlated and it has a proportional bivariate distribution ($\rho = 1$, $\sigma_1 = k\sigma_2$ and $\bar{\xi}_1 = k\bar{\xi}_2$ for $k > 0$). Flexible technology can have risk pooling value even if the product markets are perfectly positively correlated. This observation is in the spirit of Proposition 6 in Van Mieghem (1998), which is based on the price-differential of two products in a price-taking newsvendor setting. In our case, the value comes from the fact that for non-proportional bivariate distributions, the optimal production quantities with the flexible technology in stage 2 are state dependent such that there is still value from production switching at different $\boldsymbol{\xi}$ realizations.

Optimal Portfolio Choice

The cost thresholds developed in Proposition 4 reveal which technology is more profitable with and without financial risk management, but we need several more elements to fully characterize the solution to (3.3). Four more cost thresholds achieve this purpose. These thresholds are summarized in Table 2.2 and derived in the Appendix I (Chapter 5).

Threshold	**Usage**
$\bar{c}_F(c_D, \mathbf{0})$	Comparison between technologies without engaging in FRM
$\bar{c}_F(c_D, \mathbf{H}^*)$	Comparison between technologies with optimal FRM
$\bar{c}_F^S(c_D)$	Comparison between technologies with symmetric F_T and γ_T
$\underline{F}_T^{-FRM}$	Comparison between investing in T without FRM and not investing in any technology
$\underline{F}_T^{FRM}$	Comparison between investing in T with FRM and not investing in any technology
$\underline{F}_{FRM}^T$	Comparison between FRM and no FRM with technology T
$\bar{c}_T(c_{-T}, H_T^*, 0)$	Comparison between technology T with FRM and $-T$ without FRM

Table 2.2: Thresholds used in solving for the firm's optimal strategy. The first three were derived in Proposition 4 and the last four are derived in Propositions 29, 30 and 31 in the Appendix.

The "algorithm" to solve (3.3) is as follows: We use the variable cost thresholds derived in Proposition 4 to determine the optimal technologies yielding Λ_{FRM} and

Λ_{-FRM}. Using the fixed technology cost thresholds $\underline{F}_T^{-FRM}$ and $\underline{F}_T^{FRM}$, if we determine that not investing in any technology dominates either exactly one or both of Λ_{FRM} and Λ_{-FRM}, (3.3) is solved. Otherwise, we need to compare Λ_{FRM} and Λ_{-FRM}. If the same technology is optimal in both cases, then the fixed financial risk management cost threshold $\underline{F}_{FRM}^T$ is used to determine whether FRM or no FRM is optimal with that technology and (3.3) is solved. If different technologies are optimal with and without FRM, then $\bar{c}_T(c_{-T}, H_T^*, 0)$ is used to determine the optimal solution. This completes the characterization of the optimal portfolio. The next three sections highlight and discuss a series of insights that can be obtained from this analysis.

2.5 Observations Concerning the Optimal Risk Management Portfolio

In this section, we make several observations about the structure of the optimal risk management portfolio and its managerial implications. We start with an observation that illustrates the limits of the value of each risk management strategy.

Corollary 2 *If capital markets are perfect,* $\overline{F}_{FRM}^F = \overline{F}_{FRM}^D = 0$*: financial risk management has no value. If product markets are perfect, and absent a fixed cost or salvage value advantage,* $\bar{c}_F(c_D, \mathbf{H}^*) = \bar{c}_F(c_D, \mathbf{0}) = c_D$*: flexible technology has no value.*

Without capital market imperfections, the firm is not exposed to deadweight costs of external financing, as discussed in Assumption 15. In this case, financial risk management does not have any value. This is consistent with the decoupling of operational and financial decisions in perfect capital markets (Modigliani and Miller 1958). If there is no demand uncertainty ($\mathbf{\Sigma} = \mathbf{0}$), the product markets are perfect, and the firm is not exposed to supply-demand mismatch costs. Absent a fixed cost or salvage value advantage, flexible technology does not have any value. Observation 2 confirms our intuition about the risk management role of each strategy in counterbalancing the effects of costs that originate from product and capital market imperfections.

Corollary 3 *The firm can optimally speculate with forward contracts. Flexible technology can trigger speculative behavior.*

While firms frequently use financial derivatives for hedging purposes, Bodnar et al. (1998) document that some firms take speculative positions with financial derivatives. Froot et al. (1993) show that speculation may indeed be optimal when there is an external financing cost and the return on the operational investments and the risk variable are statistically correlated. They also conclude that in the absence of such correlation, the firm optimally fully hedges. In Proposition 3, we prove that the full-hedging conclusion need not hold if there are fixed costs of technology investment: Firms with limited expected internal endowment may optimally speculate to be able to invest in capacity. The majority of empirical papers assume that firms use financial derivatives for hedging purposes (Geczy et al. 1997). Observation 3 illustrates that such an assumption can be problematic in industries with fixed cost requirements.

It is interesting to note that speculation can be triggered by investment in flexible technology. The higher investment cost of flexible technology induces the firm to speculate while it uses forward contracts for hedging purposes with dedicated technology. This illustrates the intertwined nature of the integrated risk management portfolio. Engaging in operational risk management (flexible technology) may have a structural effect (going from hedging to speculation) on financial risk management.

Firms may limit their usage of financial risk management to hedging only, since speculation is typically not viewed as a desired strategy. Non-speculative use of financial risk management imposes a hedging constraint on the feasible set of forwards by imposing $H_T \geq 0$, which yields the following outcome:

Proposition 5 *If the firm uses forward contracts for hedging purposes only, then the firm optimally may not engage in financial risk management even if it is costless* ($F_{FRM} = 0$).

The intuition of this result is similar to the full speculation case above, obtained in the case of low expected internal endowment value. The firm is better off by leaving the exposure to asset price as high as possible (this corresponds to $H_T^* = 0$)

to be able to invest in capacity. Empirical studies unanimously demonstrate more widespread usage of financial risk management among large firms, and this observation is attributed to the fixed costs of establishing a financial risk management program (Allayannis and Weston 1999). Proposition 5 proposes another possible explanation: the no-speculation constraint on financial derivative usage. With this constraint, small firms (that have low internal endowments) do not engage in financial risk management.

In a recent empirical study, Guay and Kothari (2003) find no significant usage of financial risk management among non-financial firms, and suggest that these firms may be using operational hedges instead to manage their risks. We observe that indeed, firms can rely only on operational hedges in an integrated risk management framework.

Corollary 4 *Any risk management portfolio can be optimal. Financial risk management is not a panacea. Firms can rely only on flexible technology for risk management purposes.*

If financial risk management was costless, it would always be in the optimal risk management portfolio. Our analysis finds two reasons why firms may not use financial risk management: i) Its fixed cost is high. Since non-financial firms do not have as much expertise as financial firms in financial risk management, its effective fixed cost could be higher for them, which provides support for the observed difference in usage. ii) The firm limits itself to only hedging even if it is costless. Thus, not only the investment cost of financial risk management, but also the interplay between financial and operational decisions is important in determining the optimal risk management portfolio. The firm should evaluate financial risk management as an integral part of the firm's overall investment strategy. The next section provides guidelines about optimal portfolio selection.

2.6 Characteristics of the Optimal Risk Management Portfolio

In this section, we delineate the main drivers of the optimal risk management portfolio and analyze the interplay between financial and operational risk management. In §2.6.1, we relate the optimal risk management portfolio to firm, industry, technology, product market (demand variability and correlation) and capital market (external financing frictions) characteristics. We then analyze the interaction between operational and financial risk management strategies in §2.6.2. For this analysis, we proxy the firm size using the level of internal (stage 1) endowment. In particular:

Definition 1 *The firm is defined to be small (large) if the firm borrows (does not borrow) from external markets with flexible technology and full hedging,* $\omega_0^{FRM} + \overline{\alpha}_1 \omega_1^{FRM} \in \Omega_F^2(\Omega_F^0)$.

The finance literature qualitatively refers to small and large firms according to the degree to which they are affected by external financing frictions. This definition formalizes this concept in the context of our model. We parameterize the internal (stage 1) endowment as $(\lambda\omega_0, \lambda\omega_1)$ and the fixed technology costs as $F_D = F$, $F_F = F + \delta$ with $\delta \geq 0$. For tractability, we impose some parameter restrictions.

Assumption 8 *Let* $\beta = \frac{\omega_0}{\omega_0 + \alpha_0\omega_1}$, $\gamma_T = 0$, $E \geq \frac{c_T \mathbf{1}' \mathbf{K}_\mathbf{T}^{\mathbf{1}} (1-ab)}{-(b+1)a}$, $F_T \leq \overline{F}_T = \frac{c_T \mathbf{K}_\mathbf{T}^{\mathbf{1}} (1+a)}{-(b+1)a}$, *and* $F_T < \underline{F}_T^{-FRM}$.

These assumptions ensure the following: $F_{FRM} \leq \omega_0 + \alpha_0\omega_1$, so that financial risk management is feasible, and undertaking financial risk management or not can be optimal. If the firm engages in financial risk management, it optimally fully hedges; this rules out cases where the optimal financial risk management level cannot be uniquely characterized. The firm is not constrained by the credit limit, so the effective financing friction is the unit financing cost a. Finally, the optimality of not investing in either technology is ruled out.

2.6.1 Comparative Statics Results

We define Δ_T as the value of financial risk management (FRM) with technology T:

$$\Delta_T \doteq \mathbf{E}\left[\pi_T\left(B_{FRM}(\alpha_1, \omega_1^{FRM})\right)\right] - \mathbf{E}\left[\pi_T(B_{-FRM}(\alpha_1))\right]. \tag{2.5}$$

To investigate the main drivers of the optimal portfolio choice, we carry out comparative statics analysis on the variable cost thresholds $\bar{c}_F(c_D, \mathbf{H}^*)$ and $\bar{c}_F(c_D, \mathbf{0})$, and on Δ_T. The results below hold locally such that Assumption 8 and the defining regions for small and large firms are not violated.

Proposition 6 *(Technology Choice) With symmetric fixed technology costs ($F_F = F_D$), $\bar{c}_F(c_D, \mathbf{H}^*)$ and $\bar{c}_F(c_D, \mathbf{0})$ are invariant to the unit financing cost (a), the fixed costs of both technologies (F) and the internal endowment (λ) of the firm. With asymmetric fixed costs ($F_F > F_D$), $\bar{c}_F(c_D, \mathbf{H}^*)$ and $\bar{c}_F(c_D, \mathbf{0})$ decrease in the fixed costs of both technologies and the unit financing cost, and increase in the internal (stage 1) endowment of the firm.*

With symmetric fixed costs, the technology ordering is independent of financing cost, fixed costs and internal (stage 1) endowment. With asymmetric fixed costs, since flexible technology has a higher investment cost, any increase in costs (fixed cost, financing cost) favors the dedicated technology; a decrease in costs (such as an increase in the internal (stage 1) endowment), favors the flexible technology.

Proposition 7 *(Value of FRM) The value of FRM increases in the external financing cost (a) for large firms. For small firms, the value of full hedging increases (decreases) in the external financing cost at low (high) levels of F_{FRM}. For large (small) firms, the value of FRM increases (decreases) in the fixed cost of technology (F) and the demand variability (σ), and decreases (increases) in the internal (stage 1) endowment (λ) and the demand correlation (ρ).*

We now explain the drivers of Proposition 7 by grouping the results that have similar intuition. Since with Assumption 8, the firm optimally fully hedges with financial risk management, we refer to the firm engaging (not engaging) in financial risk management as the hedged (unhedged) firm.

The effect of external financing cost. Financial risk management is valuable since it reduces risk exposure and hence the expected borrowing level. At the same time, it is costly, and there is an opportunity cost for engaging in FRM: the firm may even need to borrow additional funds to finance its operational investments. These two drivers combine to determine how an increase in financing cost impacts the financial risk management decision of the firm. For large firms, the hedged firm – by Definition 1– does not borrow at all, while the unhedged firm is adversely affected from increasing financing costs. Therefore, the value of financial risk management increases in the financing cost. For small firms, this trade-off depends on the fixed cost of financial risk management. For low fixed costs, the value of financial risk management increases in financing costs; at high fixed costs, the opposite occurs.

The effect of fixed technology cost and internal (stage 1) endowment. The proof of the proposition reveals that there is one fundamental driver that explains both comparative statics results: the level of reliance on external financing, as summarized in Table 2.3. A firm's reliance on external financing increases as the fixed investment cost F increases and the internal (stage 1) endowment level λ decreases. By Definition 1, the large hedged firm does not need to borrow and the large unhedged firm borrows in some budget realizations. Therefore, increasing the reliance on external financing adversely affects the unhedged firm while not affecting the hedged firm. We conclude that for large firms, the value of FRM increases as the need for external financing increases. Since the small hedged firm, by Definition 1, always borrows and the small unhedged firm only borrows in some budget realizations, increasing the reliance on external financing adversely affects the unhedged firm, but it affects the hedged firm even more. We conclude that for small firms, the value of FRM decreases as the need for external financing increases.

The effect of demand correlation and demand variability. These two factors have an effect on the firm only with flexible technology. The proof of the proposition reveals that there is one fundamental driver that explains these two comparative

Case	Borrowing level	Increasing reliance on external financing
Large unhedged firm	Borrows in some states	Increases the value of FRM since
Large hedged firm	Does not borrow	the unhedged firm borrows more in expectation
Small unhedged firm	Borrows in some states	Decreases the value of FRM since
Small hedged firm	Borrows in all states	the hedged firm borrows more in expectation

Table 2.3: Increasing the reliance on external financing has the opposite effect on the value of financial risk management for large and small firms. A firm's reliance on external financing increases as the fixed investment cost F increases, and it decreases as the internal (stage 1) endowment level λ increases.

statics results: the marginal change in the optimal investment level with changes in these factors, as summarized in Table 2.4. A firm's optimal investment level decreases as the demand variability decreases or the demand correlation increases. The small unhedged firm borrows only in some budget realizations, while the small fully hedged firm always borrows. As a result, the small hedged firm employs a more conservative investment policy (the capacity investment level is lower at each state) than the unhedged firm since its exposure to external financing costs is higher. Consequently, a similar change in variability or correlation alters the small hedged firm's optimal investment policy to a lower extent than the unhedged firm's; its optimal investment level is more robust to changes in these factors. Therefore, while a reduction in the optimal investment level (due to a decrease in variability or an increase in correlation) adversely affects the small hedged firm, it affects the small unhedged firm even more. We conclude that for small firms, the value of FRM increases as the optimal investment level decreases. For large firms, the opposite result holds. This follows from parallel arguments based on the fact that the large unhedged firm needs to borrow in some budget realizations, while the large hedged firm does not.

Synthesis. Table 2.5 summarizes the main drivers of each optimal portfolio choice for large and small firms by combining Propositions 4, 6 and 7 for technologies with asymmetric fixed cost ($F_F > F_D$). By definition, if the variable cost thresholds increase in a parameter, flexible technology is preferred under a larger set of conditions

Case	Borrowing level	Reduction in the optimal investment level at $\tilde{B}$
Large unhedged firm	Borrows in some states	Decreases the value of FRM since the hedged firm's optimal investment is less conservative and less robust
Large hedged firm	Does not borrow	
Small unhedged firm	Borrows in some states	Increases the value of FRM since the hedged firm's optimal investment is more conservative and robust
Small hedged firm	Borrows in all states	

Table 2.4: A reduction in the optimal investment level at each state has the opposite effect on the value of financial risk management for large and small firms. A firm's optimal investment level decreases as the demand variability decreases or the demand correlation increases.

as that parameter increases, and we say that "flexible technology is favored." Similarly, if $\Delta_{T'}$ increases in a parameter, we say "financial risk management is favored." While not exact, this usage captures the direction of change. For example, high demand variability and low demand correlation favor investing in flexible technology and undertaking financial risk management for large firms. This is how Table 2.5 is constructed. We note that the capital intensity of an industry can be captured by keeping the internal endowment level constant and altering the fixed technology costs. With a given internal endowment level, a sufficiently high (low) fixed cost implies a small (large) firm according to our definition. Therefore, our results about small and large firms can be interpreted as being relevant for capital intensive and non-capital intensive industries, respectively.

The main message of Table 2.5 is that the size of the firm is key to optimal portfolio choice. As explained earlier, the three fundamental drivers behind the optimal portfolio choice (opportunity cost of financial risk management, level of reliance on external financing, and robustness of the optimal capacity investment level to variability and correlation) work in opposite directions for small and large firms. Therefore, different size firms may choose the same optimal portfolio for entirely different reasons.

Table 2.5 is for asymmetric fixed technology costs. With symmetric fixed costs, it follows from Proposition 4 that the technology ordering is independent of changes in any parameter. Therefore, changes in parameter levels only affect the choice between

Portfolio Choice	Large Firms	Small Firms
F with FRM	High demand variability Low demand correlation	High internal endowment Low technology fixed costs Low financing costs with low F_{FRM}
D with FRM	Low internal endowment High technology fixed costs High financing costs	Low demand variability High demand correlation High financing costs with low F_{FRM}
F without FRM	High internal endowment Low technology fixed costs Low financing costs	High demand variability Low demand correlation Low financing cost with high F_{FRM}
D without FRM	Low demand variability High demand correlation	Low internal endowment High technology fixed costs High financing costs with high F_{FRM}

Table 2.5: Main Drivers of the Optimal Risk Management Portfolio with Asymmetric Fixed Technology Costs.

undertaking FRM or not. Consequently, all the conditions in Table 2.5 that favor flexible or dedicated technology with FRM and without FRM for a given firm size favor using FRM and not using FRM, respectively. We conclude that the technology cost characteristic is also key to the optimal portfolio structure.

We now relate our theoretical findings to the associated empirical literature. The financial risk management literature relates the value of financial risk management to underlying exposure, growth opportunities and size of firms (Allayannis and Weston 1999). Our results demonstrate that the value of financial risk management also depends on the product market and technology characteristics, and that there are subtle differences between large and small firms.

Gay and Nam (1998) say that firms with higher investment opportunities that are exposed to higher external financing frictions and lower levels of cash make greater use of financial derivatives. We show (in the proof of Proposition 7) that the effect of cash ω_0 is the same as the effect of internal (stage 1) endowment: A lower internal (stage 1) endowment increases the value of hedging for small firms, but not for large firms. Therefore, our results support their argument for small firms, but not for large firms.

The financial risk management literature hypothesizes that the value of financial risk management increases as financing frictions increase by invoking the counterbalancing effect of financial risk management with respect to external financing frictions (Mello and Parsons 2000). Our results support this argument for large firms, but not for small firms. The key is how much the firm needs to borrow after undertaking financial risk management.

2.6.2 The Interaction of Operational and Financial Risk Management

We first investigate whether flexible technology and financial risk management are substitutes or complements in an integrated risk management framework. They are defined to be substitutes if the firm invests in flexible technology when the firm is not allowed to use financial risk management and switches to dedicated technology when the firm engages in financial risk management; they are called complements if the switch is from dedicated to flexible technology.

Proposition 8 *Flexible technology and financial risk management can be complements or substitutes. Small (large) firms tend to substitute (complement) flexible technology with financial risk management.*

The main driver of Proposition 8 is the value of financial risk management with each technology. Flexible technology is more expensive, so it is more exposed to external financing costs. The use of financial risk management allows large firms to secure a budget level sufficient to eliminate borrowing. Thus, large firms complement flexible technology with financial risk management in their integrated risk management portfolio. Small firms need to borrow to invest in flexible technology, even using financial risk management, but may not need to borrow for dedicated technology if they use financial risk management. In other words, the value of financial risk management is higher with dedicated technology. This explains why flexible technology and financial risk management are substitutes for small firms.

Interestingly, the empirical literature also finds mixed results on this question, albeit in other contexts. Geczy et al. (2000) document complementarity between operational (physical storage) and financial means of risk management among natural gas pipeline firms. In a multinational context, Allayannis et al. (2001) find that financial and operational (geographical diversification) risk management tools are substitutes. In a different framework, Chod et al. (2006b) provide another theoretical justification for these mixed empirical results by focusing on the effect of financial risk management on the optimal flexibility level of the firm. They demonstrate that financial risk management is a complement (substitute) to operational flexibility when the optimal flexibility level increases (decreases) with financial hedging.

We next analyze whether the value of operational risk management (defined as the expected (stage 0) equity value difference between flexible and dedicated technologies) is more or less robust to changes in product and capital market conditions when financial risk management is undertaken. Robust strategies are preferable because they perform well under a wider range of parameters, and can be implemented with more confidence.

Proposition 9 *For large (small) firms, the value of operational risk management is less (more) robust to changes in product market conditions* (ρ, σ) *and more (less) robust to changes in capital market conditions* (a) *with financial risk management than without.*

The proof of the proposition reveals that the robustness with respect to product market conditions is linked to the value of FRM with flexible technology. The value of operational risk management is more or less robust with respect to correlation if the value of FRM decreases or decreases in correlation, respectively. This is valid for small and large firms, respectively, as we discussed in §2.6.1. Robustness with respect to variability follows from a similar argument. Robustness with respect to the unit financing cost is determined by the difference between the value of FRM with flexible and dedicated technologies: The value of operational risk management is more robust to changes in a if the value of FRM with flexible technology increases more rapidly than the value of FRM with dedicated technology in response to an increase in a.

Proposition 9 again illustrates the intertwined nature of operational and financial risk management strategies: Engaging in financial risk management has the opposite impact on the robustness of the value operational risk management with respect to product and capital market conditions.

2.7 Value and Effect of Integrated Decision Making

Sections 2.5 and 2.6 analyzed the properties of the optimal integrated risk management portfolio and its drivers. In practice, firms may not take an integrated approach to these decisions; operational and financial risk management decisions may be taken independently. In this section, we focus on the value and effect of integrated decision making. We relax the restrictions of Assumption 8 and focus on general parameter settings.

If we ignore its effects on operational decisions, financial risk management does not have any value because forward contracts are investments with zero expected return. For this reason, we take no FRM as the non-integrated benchmark. Since the non-integrated benchmark is no FRM, the results of this section can also be interpreted as the effect of engaging in FRM on the firm's performance and optimal decisions. The effect of FRM on the optimal expected capacity investment and external borrowing level is ambiguous:

Proposition 10 *Engaging in financial risk management can increase or decrease the optimal expected capacity investment and the optimal expected borrowing levels.*

Since financing frictions negatively impact the stage 1 capacity investment level at each budget state, and the firm uses FRM to counterbalance the effect of financing frictions, one may expect that with FRM, the firm's expected borrowing level would be lower and the expected capacity investment level would be higher than without. On the other hand, if there is cost associated with engaging in financial risk management ($F_{FRM} > 0$), the firm has less internal endowment to invest in capacity at each budget state, and has to borrow additionally to compensate for F_{FRM}. In the proof of Proposition 10, we illustrate that even if FRM is costless, the optimal expected capacity investment can decrease and the expected borrowing level can decrease. This is a direct consequence of the joint optimization in external borrowing and capacity levels. The fundamental driver of this result is the marginal profit of the capacity investment in the joint optimization problem as we discussed in §3.4.2.

Proposition 10 shows the dependence of capacity investment on financial risk management. We now analyze the effect of engaging in financial risk management on the technology choice:

Corollary 5 *The firm may make different technology decisions with and without financial risk management.*

In their numerical analysis, Ding et al. (2005) demonstrate that financial risk management can alter more strategic operational decisions (global supply chain structure) than the capacity investment levels. Observation 5 is in line with their conclusion. We analytically prove that the technology choice of the firm may be altered by engaging in FRM. The direction of change in technology choice is determined by the value of FRM with each technology. Proposition 8 is an example for such changes and provides the intuition with some restrictions on the parameter levels.

The analysis above illustrates the effect of integrating risk management decisions on the firm's decisions. We now analyze the value of such integration as a function of firm size. To separate the value of integration from the cost of FRM, we use $F_{FRM} = 0$. Here, our definition of a large firm is the same as Definition 1, but our definition of a small firm is slightly more restrictive. We refer to firms with very limited expected internal endowment value that optimally fully speculate with FRM as small firms. Since under the conditions of Assumption 8, these firms fully hedge with FRM, the new definition is consistent with Definition 1 and corresponds to a subset of small firms in §2.6 that have a significantly low expected internal endowment value.

Corollary 6 *The value of integration is low for small firms with low cash levels* (ω_0) *and large firms with high cash levels. If the firm uses financial risk management only for hedging purposes, the value of integration is higher for large firms than for small firms.*

The value of integration is equivalent to the value of engaging in FRM. Since large firms with high cash levels are not significantly exposed to external financing frictions without FRM, the value of FRM, and hence the value of integration is low. In the extreme case, a cash level sufficient to finance the budget-unconstrained optimal investment level completely removes the exposure to external financing frictions and FRM has no value. For small firms with low levels of cash, the additional benefit of full speculation ($H_T^* = \frac{\omega_0}{\alpha_1}$) over not using FRM ($H_T^* = 0$) is low. In the extreme case, if the small firm does not have any cash ($\omega_0 = 0$), then FRM has no value.

When the firm uses financial risk management only for hedging purposes, it follows from Proposition 5 that small firms optimally do not engage in FRM. In this case, integration has no value. Large firms generally fully hedge with FRM, therefore integration has value for them. In a numerical analysis not reported here, we observe a similar pattern without imposing the hedging constraint.

2.8 Robustness of Results to Model Assumptions

In this section, we investigate the robustness of our results to the assumptions presented in §3.3.

Non-identical and exogenous financing costs. We assumed a unique external financing cost structure (a, E). The firm can be exposed to a different external financing cost structure (a_T, E_T) with each technology $T \in \{D, F\}$. All the analytical results of §2.4 continue to hold by replacing (a, E) with (a_T, E_T) where a lower unit borrowing cost is associated with a higher credit limit. The main insights of the paper do not change except that the technology with lower a_T and higher E_T is favored in the optimal risk management portfolio.

Endogenous financing costs. In this paper, we focus on a partial equilibrium setting where the financing costs are exogenous and identical for each technology. In a general equilibrium setting, the financing cost for each technology is determined by the interaction between the firm and a creditor. In Boyabatlı and Toktay (2006), we derive the equilibrium level of secured loan commitment contracts (a_T^*, E_T^*) for each technology in a creditor-firm Stackelberg game using a similar firm model. We show that the borrowing terms will be independent of technology choice when the creditor has limited information about the firm and the technologies, there is no credible way of information transmission, and the creditor bases its assessment of default probability on the same cash flow distribution of the firm for any technology. These conditions are relevant for bank financing where banks rely on the credit history of the firm for credit risk estimation and do not have operational expertise. All of the results in this paper are valid in the general equilibrium sense under these conditions. We refer the reader to the next chapter for a detailed treatment of endogenous financing costs.

Unsecured loan commitment contracts. If the firm uses unsecured loan commitment contracts ($P = 0$), the firm only receives the salvage value of the non-pledgable technology in the default states. The limited liability of the shareholders left-censors the stage 2 equity value distribution at 0. The expected (stage 1) equity value is calculated using conditional expectations with respect to default and non-default events. The probability of default depends on the capacity investment level, external borrowing level and the risk-pooling value of the technology choice. At stage 1, similar to secured lending, the firm optimally borrows so as to finance the optimal capacity investment level. In a single-product price-taking newsvendor setting, Babich et al. (2006) provide conditions under which the expected (stage 1) equity value is unimodal (though not concave) in capacity. With two products and endogenous pricing, the optimal capacity investment level is very hard to solve and becomes intractable for flexible technology because of the dependence on default regions with bivariate product market uncertainty. In our paper, the effect of limited liability is inherent in the financing cost structure (a, E). When the capital market imperfection costs are default-related (e.g. bankruptcy costs), if there were no limited liability then the creditor would be sure to recoup the face value of the loan and default-related costs from the shareholders' personal wealth. With such a riskless loan, the cost of the loan would be the risk-free rate ($a = 0$) and the firm could raise sufficient funds to finance the budget unconstrained capacity level ($E = P$).

If we allow unsecured lending in our setting, we conjecture that the optimal capacity investment level would be lower: The marginal cost of borrowing is less than $1 + a$ because of the default, which should induce the firm to borrow more and invest more in capacity. Structural results related to financial risk management are expected to hold. How the technology choice would change is not clear because of the dependence on default regions. The arguments in this section are also relevant for i) partially secured lending (P is positive but not sufficient to finance the budget unconstrained capacity investment), and ii) secured lending with default-related costs deducted from the firm's seized assets by the creditor in the case of default.

Positive production cost at stage 2. Let y denote the unit production cost for both products with either technology. With $y > 0$, the optimal production vector at stage 2 is limited by the cash availability of the firm in addition to the physical capacity constraints. In this case, the literature often uses a clearing-pricing strategy for tractability that fully utilizes the physical capacity (see for example, Chod and Rudi 2005). If we assume a clearing-pricing strategy, the firm optimally borrows so as to fully utilize the physical resource in stage 2 and all the results of our paper continue to hold by replacing c_T with $c_T + y$.

If we focus on the optimal pricing policy with $y > 0$, the optimal production vector with flexible (dedicated) technology is state dependent and has a complex form that is characterized by a two-region (six-region) partitioning of the demand space (ξ_1, ξ_2) with respect to capacity constraints[3]. The optimal capacity level is lower than the $y = 0$ case, and accounts for the state-dependent optimal production vector. With flexible technology, the firm optimally borrows the exact amount required for the full utilization of the physical resource. With dedicated technology, the optimal borrowing level is such that the physical resources are never fully utilized. Financial capacity has a risk-pooling benefit with dedicated technology because the firm can allocate the financial resource to each physical capacity contingent on the demand realization. Because of this additional risk-pooling benefit of dedicated technology, flexible technology is more adversely affected from $y > 0$ compared to $y = 0$. With $y > 0$, the majority of the insights and the structural results obtained with $y = 0$ remain valid. The results concerning the product market characteristics (ρ, σ) are among the few exceptions. Similar to flexible technology, the value of dedicated technology decreases in ρ and increases in σ. This is a direct consequence of the declining risk-pooling value of the financial capacity. The optimal technology choice as a function of product market conditions is not clear in this setting.

Seizable salvage value of technology. We assume that the creditor cannot seize the salvage value of the technology in case of default. If the salvage value of the technology is offered as an additional collateral, then the creditor can seize the technology. With exogenous financing costs, seizable technology does not have any impact on the results of this paper. With endogenous financing costs and immediate liquidation of technology, collateralizing the technology reduces the default risk and hence external financing costs in equilibrium. Different salvage values of the technologies have a significant impact on the technology choice in equilibrium.

Fixed cost of technology is incurred at stage 0. If the firm incurs the fixed cost of technology at the time of commitment (at stage 0), then this fixed cost is deducted from the firm's internal stage 1 endowment (ω_0, ω_1) in the same way as F_{FRM}. With this assumption, the firm always optimally fully hedges with financial risk management; hence Observation 3 and Proposition 5 do not hold. All the other results remain valid. The same conclusions hold in the absence of technology fixed costs $(F_F = F_D = 0)$.

2.9 Conclusions

This paper analyzes the integrated operational and financial risk management portfolio of a firm that determines whether to use flexible or dedicated technology and whether to undertake financial risk management or not. The risk management value of flexible technology is due to its risk pooling benefit under demand uncertainty. The financial risk management motivation comes from the existence of deadweight costs of external financing. Financial risk management has a fixed cost, while technology investment incurs both fixed and variable costs. The firm's limited budget, which depends partly on a tradable asset, can be increased by borrowing from external markets, and its distribution can be altered via financial risk management.

In a parsimonious model, we solve for the optimal risk management portfolio, and the related capacity, production, financial risk management and external borrowing levels, the majority of them in closed form. We characterize the optimal risk management portfolio as a function of firm size, technology and financial risk management costs, product market (demand variability and correlation) and capital market (external financing costs) characteristics.

We find that three fundamental drivers explain the optimal portfolio choice: the robustness of the optimal capacity investment with respect to product market characteristics, the level of reliance on external financing and the opportunity cost of financial risk management. Our results provide managerial insights about the design of integrated operational and financial risk management programs. A firm that operates in highly variable or highly negatively correlated product markets should use flexible technology with financial risk management if the firm has sufficiently high internal endowment (large firm); and without financial risk management if the firm has limited internal endowment (small firm). For large firms with low (high) external financing costs, flexible technology with financial risk management (dedicated technology without financial risk management) is the best risk management portfolio. For small firms, the insights related to technology choice under high and low external financing costs continue to hold but the firm should only use financial risk management if the fixed cost of financial risk management is sufficiently low.

Our analysis clearly shows the intertwined nature of operational and financial risk management strategies and illustrates their subtle interactions. For example, operational and financial risk management can be complements or substitutes depending on the firm size. Flexible technology and financial risk management tend to be substitutes for small firms and complements for large firms. The fundamental driver of this result is the difference in the value of financial risk management with each technology. We also show that the firm's use of financial instruments for speculative reasons can be triggered by choosing the higher cost flexible technology.

Our analysis extends the modelling framework of Froot et al. (1993) by formalizing operational investments and imposing a cost for financial risk management. With our more detailed operational model, some of their findings do not continue to hold. For example, firms can optimally use financial risk management for speculative purposes even if the returns from operational investments are independent from the financially hedgable risk variable. The driver of this result is the fixed cost of technology. In addition, we show that firms may choose not to use financial risk management due to its cost when resources are limited. The effective cost of financial risk management is larger than its fixed cost because of the existence of operational investments: After incurring the fixed cost of financial risk management, the firm may need to borrow additional funds to finance its operational investments, which imposes an opportunity cost on the firm. These results enhance our understanding of the effect of operational factors in risk management and underline the importance of integrated decision making.

This paper brings constructs and assumptions motivated by the finance literature into a classical operations management problem. In turn, we provide theoretical support for some observations made in the empirical finance literature and highlight additional trade-offs in some others. For example, we establish that the value of financial risk management increases in external financing costs only for large firms and not for small firms. This is in contrast to the existing understanding that this is true for any firm. There is evidence that large firms use financial instruments more frequently than small firms. This observation is attributed to the fixed cost of establishing a financial risk management program. Our analysis proposes another explanation that is based on the hedging constraint sometimes imposed in practice: If firms are allowed to use financial instruments for hedging purposes only, it is optimal for small firms to not undertake financial risk management even if it is costless.

Our paper opens new empirical avenues. The existing literature on risk management typically does not capture operational aspects such as characteristics of different technologies and product market characteristics. As demonstrated by our analysis, these can have a significant effect on the risk management portfolio and generally have opposite effects for large and small firms. The distinction we make between large and small firms (or equivalently, between capital intensive and non-capital intensive industries), and our results related to the effect of technology and product market characteristics on the risk management portfolio provide new hypotheses that can be tested empirically. For example, we expect to see that large firms engage in financial risk management less frequently than small firms in highly positively correlated markets. We also expect to see a positive relation between fixed technology costs and the frequency of engaging in financial risk management for large firms and a negative relation for small firms.

In §2.8, we discussed the implication of relaxing some of our assumptions. Other interesting research directions remain. For example, this paper focuses on a monopolistic firm. In an integrated risk management framework, strategic risk management has not received much attention. Goyal and Netessine (2005) analyze the value of flexible technology under product market competition. It would be interesting to incorporate financial risk management decisions of the firm in this competitive setting. The financially hedged firm may invest in more costly flexible technology whereas the non-hedged competitor may not because of external financing frictions. Financial risk management will certainly have a non-trivial impact on the equilibrium of the game. Dong et al. (2006) take a step in this direction by modeling operational flexibility and financial risk management decisions of a global firm facing a local competitor that can only respond by setting its production quantity.

We assume an exogenous external financing cost structure. Technology characteristics can affect the external financing costs in equilibrium; this occurs if the lender has information about the firm's technology options and the ability to assess their operational and collateral value. In this case and with loan commitment contracts, the financing cost structure would depend on the firm's likelihood of borrowing and the default risk conditional on the borrowing level. Flexible technology has higher costs, and requires more external borrowing than dedicated technology; but the risk-pooling value of flexible technology decreases the default risk. The different collateral values of each technology bring another facet to this interaction. It is interesting to analyze which effect dominates under what conditions. The broader question is whether firms should use flexible versus dedicated technology in imperfect capital markets. We analyze these issues in a companion paper (Boyabatlı and Toktay 2006b).

Notes

[1]With the exception of sensitivity results with respect to demand variability and correlation: These results require formalization of demand variability and correlation via specific distributional or structural (using stochastic orderings) assumptions.

[2]To capture the effect of demand correlation and variability, we use different measures that are commonly used in the literature. Throughout the paper, by "an increase in demand variability," we refer to any one of the following cases: i) $\boldsymbol{\xi}$ has a symmetric bivariate lognormal distribution and σ monotonically increases, ii) $\boldsymbol{\xi}$ with independent marginal distributions is replaced with $\boldsymbol{\xi}$ with independent marginal distributions such that $\overline{\boldsymbol{\xi}} = \overline{\boldsymbol{\xi}}$ and ξ_i' is stochastically more variable than ξ_i for $i = 1, 2$, or iii) $\boldsymbol{\xi}$ with $\sigma = 0$ is replaced with $\boldsymbol{\xi}$ with $\sigma \neq 0$. By "an increase in demand correlation," we refer to any one of the following cases: i) $\boldsymbol{\xi}$ has a bivariate lognormal distribution and ρ monotonically increases, ii) $\boldsymbol{\xi}$ is replaced with $\boldsymbol{\xi}$ which dominates $\boldsymbol{\xi}$ according to the concordance ordering, or iii) $\boldsymbol{\xi}$ with $\rho \neq 1$ is replaced with $\boldsymbol{\xi}$ with $\rho = 1$. The details of the analysis can be found in the proof.

[3]The proofs for the stage 2 optimal production vector for each technology with $y > 0$

are available upon request.

Chapter 3

Capacity Investment in Imperfect Capital Markets: The Interaction of Operational and Financial Decisions

3.1 Introduction

Capacity investment is subject to internal or external financing frictions, especially in capital-intensive industries. However, as highlighted by Van Mieghem (2003, p. 275) "stochastic capacity models assume (often implicitly) either perfect capital markets, so that frictionless borrowing is possible, or that the investment size is relatively small, so that it can be internally financed without material impact on the overall valuation of the firm." The objective of this paper is to increase our understanding of how capital market imperfections affect technology choice and capacity investment. Our main contributions are: (i) to analyze the effect of capital market imperfections on capacity investment and characterize previously undocumented trade-offs that arise in imperfect capital markets; (ii) to demonstrate that these trade-offs may change traditional insights concerning capacity investment derived under the perfect market assumption; and (iii) to underline the importance of the integration of operational and financial decisions.

To this end, we model a budget-constrained manufacturer who produces and sells two products. Product demands are price-dependent, random and correlated. The firm chooses between flexible and dedicated technologies that incur variable investment costs, and determines the capacity level and the production quantities with the chosen technology. The firm's limited budget partially depends on a perfectly tradable asset. Thus, the firm is exposed both to product market (demand) and financial market (asset price) risk. The firm can relax its budget constraint by borrowing from a creditor. To capture capital market imperfections, we assume that the creditor incurs a fixed cost of bankruptcy if the firm defaults on the loan, and imposes an underwriting fee. The firm can use forwards written on the asset price to alter its budget distribution so as to counterbalance the effect of external financing costs arising from capital market imperfections.

We derive the optimal technology, capacity, production, external borrowing and financial risk management decisions of the firm and the creditor's optimal contracting decision in equilibrium. Based on these results, we show how capital market imperfections affect the operational decisions of the firm. This comparison is made possible by the existence of a natural perfect-market benchmark in our framework. In particular, we answer the following research questions:

1. How do capital market imperfections affect capacity investment and operational performance?

2. For a given technology, what are the main drivers of capacity investment level and operational performance in imperfect capital markets?

3. What are the main drivers of technology choice in imperfect capital markets?

4. Do these drivers differ from those in perfect capital markets and if so, what explains the difference?

5. What is the value of financial risk management in the creditor-firm interaction?

We demonstrate that an increase in capital market imperfection costs decreases the operational performance and the optimal capacity investment of the firm. This is because higher imperfection costs lead to higher financing costs in equilibrium.

In perfect capital markets, for a firm selling a single product, or two products using dedicated technology, the firm's optimal capacity investment level and its operational performance only depend on the expectation of demand and not its variability. In imperfect capital markets, other factors also matter. In the single product setting, the firm's optimal capacity investment level and its operational performance *decrease* in demand variability. This is because higher variability increases the default risk of the firm and induces the creditor to charge a higher financing cost in equilibrium. In the two-product setting with dedicated technology, the firm's optimal capacity investment level and operational performance also *decrease* in demand correlation. This is because the two-market investment generates diversification benefits for the firm in reducing the default probability. An increase in correlation decreases the diversification benefit, increases the default risk of the firm and, in turn, increases the equilibrium level of financing costs.

Flexible technology incurs a higher investment cost and exhibits production-switching capability. For a given capacity investment level, production switching capability is beneficial. Production-switching capability also alters the optimal capacity investment level of the firm. In perfect capital markets, these two direct effects create positive (static) value (and are traded off against the higher cost of flexible technology in determining the optimal technology choice). In imperfect capital markets, production switching capability also has strategic value through the indirect effect on the equilibrium financing cost: With identical technology costs, the expected value of production switching for a given capacity investment level acts to decrease the default risk of the firm and hence the equilibrium financing cost. However, the optimal adjustment of the capacity investment level acts to increase the expected borrowing level and the default risk of the firm, and may decrease or increase the equilibrium financing cost. The higher investment cost of flexible technology also has an effect on these static and strategic values. Putting these effects together, we show that the strategic value of production switching can be negative and this negative strategic value that only exists in imperfect capital markets can direct the firm towards dedicated technology in these markets.

Our analysis illustrates the value of financial risk management in creditor-firm interaction. In perfect capital markets, financial risk management does not have any value for the firm and does not affect the creditor's returns. In imperfect capital markets, financial risk management has both static and strategic values. For a given financing cost scheme, engaging in financial risk management has a positive value for the firm because it decreases the expected borrowing level of the firm. The strategic value comes from the effect on the creditor's expected returns. We demonstrate that the firm's engagement in financial risk management may have negative strategic value. The reduction in the firm's borrowing level decreases the expected returns of the creditor and the creditor increases the financing cost in equilibrium.

With these results, we contribute to the growing operations management literature that incorporates financial considerations in operational decision making. We provide managerial insights about technology and capacity investment for financially constrained firms that are exposed to capital market imperfection costs. In the next section, we provide more detail about how our work contributes to the existing literature. In §3.3, we describe the model and discuss the basis for our assumptions. §3.4 and §3.5 analyze the optimal strategy of the firm and the creditor, respectively. We provide a perfect market benchmark in §3.6. Our main results and contributions are provided in Sections 3.7 and 3.8 where we distill the effect of capital imperfections on the firm's decisions and performance. §3.7 examines this effect in a single-product setting. §3.8 extends this analysis to the two-product setting and investigates technology choice in imperfect capital markets. §3.9 concludes.

3.2 Literature Review

In this section, we review the streams of literature related to our paper and delineate our contributions to each stream. The stochastic capacity investment literature analyzes the value of resource flexibility in a variety of models. We refer readers to Van Mieghem (2003) for an excellent review. As highlighted in this review paper, the operations management literature (often implicitly) assumes that capital markets are perfect, in which case operational and financial decisions decouple (Modigliani and Miller 1958). In practice, capital market imperfections such as agency costs, taxes, underwriter fees and bankruptcy costs exist (Harris and Raviv 1991) and impose deadweight costs of external financing, leading operational and financial decisions to interact with each other. There is a growing body of work in operations and finance that analyze these interactions. Our paper's overall contribution to this literature is i) increasing our understanding of the effect of capital market imperfections on stochastic capacity investment; ii) demonstrating heretofore undocumented tradeoffs that arise in imperfect capital markets; iii) delineating the interaction between operational and financial decisions in capacity investment context.

In the Operations Management literature, Lederer and Singhal (1994), Buzacott and Zhang (2004), Xu and Birge (2004), Babich and Sobel (2004) and Babich et al. (2006) focus on the joint financing and operating decisions of the firm. We compare our results to two of these papers in particular. Lederer and Singhal (1994) study the joint financing (optimal mix of debt and equity) and capacity investment problem in a multi-period setting and show how the technology choice of the firm is related to its financing decision. In a numerical example, they show that the production switching value of flexible technology is even stronger with external financing because it decreases the firm's default risk by reducing the variability of cash flows. We analytically demonstrate that their result may not hold in general. Drivers of the production switching value create strategic effects on the equilibrium level of financing costs; these strategic effects can be negative and induce the firm to prefer dedicated technology.

Xu and Birge (2004) analyze the effect of taxes and bankruptcy costs on the firm's joint financing and operating decisions in a single-period single-product capacity investment setting. They demonstrate the value of integrated decision making and analyze the effect of demand variability and some other operational characteristics in imperfect capital markets. Our work is complementary to theirs. We provide analytical proofs for some of their numerical observations and extend the interaction analysis to the two-product setting. In particular, we analyze the effect of technology choice and financial risk management on the creditor-firm interaction.

Several finance papers also investigate the interaction of financing and operating decisions. Dotan and Ravid (1985) and Dammon and Senbet (1988) are examples of early studies that demonstrate the effect of operational investments on the financing policy of the firm in a single-period setting. We refer the reader to Childs et al. (2005) for a recent review of papers in this stream. More recently, a number of papers (Mauer and Triantis 1994, Mello et al. 1995, and Mello and Parsons 2000) analyze the effect of various forms of operational flexibility (e.g. shutting down the production plant) on the joint operational and financing decisions of firms in the contingent claims framework. The focus of these papers is on the financing policy of the firm with strong modeling assumptions concerning the firm's operations. As highlighted in MacKay (2003), without agency cost concerns, operational flexibility has a positive strategic effect: Operational flexibility decreases the firm's default risk by generating higher returns due to its option value and this decreases the financing cost in equilibrium. We demonstrate that this argument may not hold in general with a stronger formalization of the firm's operations. Anticipating the option value of operational flexibility (flexible technology in our case), the firm optimally adjusts other operational decisions (capacity investment and production quantity). As a result, the firm's default risk may increase, yielding a net negative strategic effect.

Our work is also related to the recent stream of papers that analyze the interaction of operational and financial decisions from an integrated risk management point of view. Zhu and Kapuscinski (2004), Ding et al. (2005), Chod et al. (2006), Dong et al. (2006) are in this stream. We refer the reader to our companion paper, Boyabatlı and Toktay (2006a) for a more detailed review of this literature. In Boyabatlı and Toktay (2006a), we use a similar, more general model of the firm, but take external financing costs to be *identical and exogenous* for each technology. This paper formalizes the capital market imperfections and *endogenizes* the external financing costs in a creditor-firm interaction. We provide new insights and undocumented trade-offs that arise from this strategic interaction.

Several papers in the finance literature empirically (e.g. Gay and Nam 1998, Geczy et al. 1997, Haushalter 2000) and theoretically (e.g. Froot et al. 1993, Smith and Stulz 1985, Leland 1998) analyze the value of financial risk management under borrowing frictions where the operating cash flows are correlated with a financially tradable index. The majority of these papers document the static value of financial risk management: With a given financing cost, the firm can use financial instruments to engineer its internal cash flows to reduce the dependence on external borrowing. A few papers (Smith and Stulz 1985, Leland 1998) demonstrate the positive strategic value of financial risk management with endogenous financing costs. Financial risk management enables the firm to reduce its default risk after the loan is taken and decreases the financing costs in equilibrium. We contribute to this literature by showing the negative strategic value of financial risk management. The main driver of this result is that in our setting financial risk management is effective before the loan is taken. Financial risk management reduces the expected borrowing level of the firm, and this may induce the creditor to charge higher financing costs in equilibrium to generate sufficient returns.

3.3 Model Description and Assumptions

We consider a creditor-firm strategic interaction where the creditor is the Stackelberg leader who determines the borrowing terms. The firm is a monopolist that sells two products in a single selling season under demand uncertainty. Differing from the majority of traditional stochastic technology and capacity investment problems, we model the firm as being budget constrained, where the budget partially depends on a hedgeable market risk. This firm can undertake financial risk management to hedge this risk. The firm chooses the technology (dedicated versus flexible), and the borrowing, financial risk management, capacity investment, and production levels so as to maximize expected shareholder wealth. After operating profits are realized, the firm pays back its debt; default occurs if it is unable to do so.

We model the firm's decisions as a three-stage stochastic recourse problem under financial market and demand risk. In stage 0, the firm decides its technology choice (flexible or dedicated), and the financial risk management level under demand and financial market risk. In stage 1, the financial market risk is resolved and the financial risk management contract (if any) is exercised; these two factors determine the internal cash level of the firm. The firm then determines the level of external borrowing and makes its capacity investment using its total budget (internal cash and borrowed funds). In stage 2, demand uncertainty is resolved and the firm chooses the production quantities for each product. Subsequently, the firm either pays back its debt or defaults. In the remainder of this section, we define the creditor's and firm's objectives and discuss the assumptions concerning each decision epoch in detail.

Assumption 9 *The creditor is risk neutral and chooses the borrowing terms to guarantee a given expected return* $U \geq 0$.

The risk-neutrality of the creditor can be justified on the basis of the fact that banks have large diversified portfolios and they are approximately risk-neutral with respect to individual loans with small risks (Rochet and Frexias 1997, p.94). U represents the reservation utility of the creditor. In the financial economics literature, the common assumption is to have perfectly competitive loan markets such that the creditor makes zero-expected profit, i.e. $U = 0$ (see, for example, Melnik and Plaut 1986). We allow strictly positive values of U which can be interpreted as expected underwriter fees. $U > 0$ is one of the capital market imperfections in our model. We discuss the implications of $U > 0$ in §3.6.

Assumption 10 *The firm maximizes the expected shareholder wealth by maximizing the expected (stage 0) value of equity. The shareholders are assumed to be risk-neutral and the risk-free rate* r_f *is normalized to 0. Shareholders have limited liability.*

The main goal of corporations is to maximize shareholder wealth. The expected shareholder wealth is a function of the expected cash flows to equity of the firm and the required rate of return of the shareholders. By assuming the risk neutrality of shareholders, we focus on maximizing the expected equity value of the firm. The required rate of return is the risk-free rate, which is normalized to 0 by assumption.

3.3.1 Stage 0

In this stage, the creditor offers its borrowing terms. The firm then determines its technology choice $T \in \{D, F\}$, and the financial risk management level H_T under financial market and demand uncertainty. The flexible technology (F) has a single resource that is capable of producing two products. The dedicated technology (D) consists of two resources that can each produce a single product. Assumptions 11-14 summarize modelling choices related to these decisions.

Assumption 11 *The creditor offers separate secured loan commitment contracts for each technology and incurs fixed bankruptcy cost BC if the firm defaults on its loan. Technology choice is verifiable. The creditor has full information about the firm.*

Loan commitment is a promise to lend up to a pre-specified amount at a pre-specified terms. In practice, most short-term industrial and commercial loans in the US are made under loan commitment contracts (Melnik and Plaut 1986). When the firm agrees to a contract at stage 0, this means that it owns the right to a loan contract that can be exercised in stage 1. If the firm defaults on its loan, the creditor incurs the fixed bankruptcy cost BC. Bankruptcy cost is the other form of market imperfection in our model. This cost includes the administrative and legal fees (Altman 1980) of bankruptcy and is often used in the literature to represent default related capital market imperfections (e.g. Smith and Stulz 1985). We discuss the characteristics of the loan commitment contract in Assumption 15 of stage 1. The contracts are finalized before the firm commits to the technology and financial risk management level. It follows from the verifiability assumption that the creditor offers technology-specific contracts.

The information endowment of the creditor is an important determinant of external financing costs. The degree of information available to the creditor depends on several factors such as the source of borrowing (e.g. bank financing, vendor financing), the extent of interaction between the creditor and the firm (credit history), the available information about the firm's financial status (e.g. credit rating, stock price) and the industry characteristics (e.g. industry reports) in which the firm is operating. In this paper, we focus on one end of the spectrum where the creditor has full information about the firm. The full-information case may represent a vendor that finances its subsidiary or the financial partner of a firm (GM Acceptance Corporation) that provides financial services to the parent company (GM). In reality, most industrial and commercial firms occasionally lend money to their customers (Rochet and Frexias 1997). With full-information, the creditor can anticipate the optimal decisions of the firm.

Assumption 12 *Technology T has variable (c_T) capacity investment costs.*

Assumption 13 *At stage 0, the firm has rights to a known internal stage 1 endowment (ω_0, ω_1). Here, ω_0 represents the cash holdings and ω_1 represents the asset holdings of the firm. The asset is a perfectly tradeable asset that has a known stage 0 price of α_0 and random stage 1 price of α_1. The random variable α_1 has a continuous distribution with positive support and bounded expectation $\overline{\alpha}_1$.*

With this assumption, in stage 0, the firm knows that the value of its endowment will be $\omega_0 + \alpha_1\omega_1$ in stage 1, where α_1 is random; this is the financial market risk in our model. This representation is consistent with practice: In general, firms hold both cash and tradable assets on their balance sheet, such as a multinational firm that has pre-determined contractual fixed payments denominated in both domestic and foreign currency, or a gold producer that produces a certain level of gold that is exposed to gold price risk. In these examples, the asset price α_1 represents the exchange rate and the gold price in stage 1, respectively. Although the cash and the asset holdings are certain, the price of the asset makes the stage 1 value of the internal endowment random. The firm can use financial risk management tools to alter the distribution of this quantity.

Assumption 14 *The firm uses forward contracts written on asset price α_1 to financially manage the market risk. Forward contracts are fairly priced. We restrict the number of forward contracts H_T such that the firm does not default on its financial transaction in stage 1.*

Forward contracts are the most prevalent type of financial derivatives used by non-financial firms (Bodnar et al. 1995). The fair-pricing assumption ensures that the firm can only affect the distribution of its budget in stage 1 – and not its expected value – by financial risk management. We restrict the feasible set of forwards to the range $\left[-\frac{\omega_0}{\alpha_1}, \omega_1\right]$. Within this range of forwards the firm never defaults on its financial transaction in stage 1. This ensures that we can use default-free prices in forward transactions.

3.3.2 Stage 1

In stage 1, the market risk α_1 is resolved. The value of the firm's internal endowment and the exercise of the financial contract (if any) determine the firm's budget B. In this stage, the firm can raise external capital if the budget is not sufficient to finance the desired capacity investment. The firm determines the amount of external borrowing and the capacity investment level under demand uncertainty.

Assumption 15 *With the loan commitment contract, the firm can borrow up to credit limit E_T from a unit interest rate of $a_T \geq r_f = 0$ with each technology $T \in \{D, F\}$. The firm has physical assets of value P (e.g. real estate) that are pledged to the creditor as collateral. We assume that at a given unit cost a_T, the creditor chooses the credit limit that can be secured with the collateral value P, $E_T(a_T) = \frac{P}{1+a_T}$. The physical assets are illiquid; they can only be liquidated with a lead time. The value of the physical assets P is sufficient to finance the budget-unconstrained optimal capacity investment level of the firm. There is a fixed bankruptcy cost BC for the creditor if the firm defaults. The creditor incurs this cost as an out-of-pocket fee and does not deduct it from the seized assets of the firm.*

We assume that the loan commitment is fully collateralized by the firm's physical assets P, i.e. $E_T(1 + a_T) = P$, since most bank loans are secured by the company's assets (Weidner 1999) and modeled as such (Mello and Parsons 2000). Although the loan is fully collateralized, if the firm's final cash position is not sufficient to cover the face value of the debt, the firm cannot immediately liquidate the collateral assets to repay its debt since the physical assets are illiquid. Under limited shareholder liability, this leads to default, in which case the creditor can seize these physical assets, liquidate them and use their liquidation value to recover the loan. With our assumption that it is incurred by the creditor as out-of-pocket fees in the default states, this cost is effectively charged to the firm ex-ante in the equilibrium unit borrowing cost a_T. This assumption is made for analytical convenience. We can show that the equilibrium level of unit financing cost decreases if the creditor can deduct the fixed cost of the bankruptcy from the seized assets[1]. This demonstrates that the firm is penalized ex-ante (before borrowing) through a more severe loan contract for not being responsible for the fixed cost of bankruptcy after default.

This external financing cost structure provides a parsimonious model that is consistent with real-life practices; allows us to capture capital market imperfections and enables us to preserve tractability.

3.3.3 Stage 2

In this stage, demand uncertainty is resolved. The firm then chooses the production quantities (equivalently, prices) to satisfy demand optimally. If the firm is able to repay its debt from its final cash position, it does so and terminates by liquidating its physical assets. Otherwise, default occurs. In this case, because of the limited liability of the shareholders, the firm goes to bankruptcy. The cash on hand and the ownership of the collateralized physical assets are transferred to the creditor. The firm receives the remaining cash after the creditor covers the face value of the debt from the seized assets of the firm.

[1]The proof is available upon request

Assumption 16 *Price-dependent demand for each product is represented by the iso-elastic inverse-demand function* $p(q_i; \xi_i) = \xi_i q_i^{1/b}$ *for* $i = 1, 2$*. Here,* $b \in (-\infty, -1)$ *is the constant elasticity of demand, and* p *and* q *denote price and quantity, respectively.* ξ_i *represents the idiosyncratic risk component.* (ξ_1, ξ_2) *are correlated random variables with continuous distributions that have positive support and bounded expectation* $(\bar{\xi}, \bar{\xi})$ *with covariance matrix* Σ*, where* $\Sigma_{ii} = \sigma^2$ *and* $\Sigma_{ij} = \rho\sigma^2$ *for* $i \neq j$ *and* ρ *denotes the correlation coefficient.* (ξ_1, ξ_2) *and* α_1 *have independent distributions. The marginal production costs of each product at stage 2 are 0.*

We make specific assumptions about the distribution of $\boldsymbol{\xi}$ throughout the text whenever necessary.

3.4 Analysis of the Firm's Problem

3.4.1 Stage 2: Production Decision

In this stage, the firm observes the demand realization $\tilde{\boldsymbol{\xi}}$ and determines the production quantities $\mathbf{Q_T}' = (q_T^1, q_T^2)$ within the existing capacity limits to maximize the stage 2 equity value.

Proposition 11 *The optimal production quantity vector in stage 2 with technology* $T \in \{D, F\}$ *for given* $\mathbf{K_T}$ *and* $\tilde{\boldsymbol{\xi}}$ *is given by*

$$\mathbf{Q_D^*} = \mathbf{K_D}, \quad \mathbf{Q_F^*} = \frac{K_F}{\tilde{\xi}_1^{-b} + \tilde{\xi}_2^{-b}} \, \tilde{\boldsymbol{\xi}}^{-b}.$$

Proof All proofs are relegated to Appendix II (Chapter 5). ■

Since the unit production cost is zero, the firm optimally utilizes the entire available capacity. With dedicated technology, the optimal individual production quantities are equal to the available capacity levels for each product. With flexible technology, the firm allocates the available capacity K_F between each product in such a way that the marginal profits for each product are equal.

3.4.2 Stage 1: Capacity Choice and External Financing

In this stage, the firm exercises the forward contract H_T and observes the asset price $\tilde{\alpha}_1$. With fair pricing, the strike price of the forward is equal to $\overline{\alpha}_1$. The stage 1 budget is therefore $B(\tilde{\alpha}_1, H_T) \doteq \omega_0 + \tilde{\alpha}_1(\omega_1 - H_T) + \overline{\alpha}_1 H_T$. We henceforth suppress $\tilde{\alpha}_1$ and H_T and denote the available budget realization by $\tilde{B} \in [0, \infty)$. For given $\tilde{B}$ and T, the firm determines the optimal capacity investment level $\mathbf{K}^*_\mathbf{T}(\tilde{B})$ and the optimal external borrowing level $e^*_T(\tilde{B})$.

Proposition 12 *The optimal capacity investment vector* $\mathbf{K}^*_\mathbf{T}(\tilde{B})$ *and the optimal external borrowing level* $e^*_T(\tilde{B})$ *for technology* $T \in \{D, F\}$ *with a given budget level* $\tilde{B}$ *are*

$$\mathbf{K}^*_\mathbf{T}(\tilde{B}) = \begin{cases} \mathbf{K}^\mathbf{0}_\mathbf{T} & \text{if } \tilde{B} \in \Omega^0_T \doteq \{\tilde{B} : \tilde{B} \geq c_T \mathbf{1}'\mathbf{K}^\mathbf{0}_\mathbf{T}\} \\ \overline{\mathbf{K}}_\mathbf{T} & \text{if } \tilde{B} \in \Omega^1_T \doteq \{\tilde{B} : c_T \mathbf{1}'\mathbf{K}^\mathbf{1}_\mathbf{T} \leq \tilde{B} < c_T \mathbf{1}'\mathbf{K}^\mathbf{0}_\mathbf{T}\} \\ \mathbf{K}^\mathbf{1}_\mathbf{T} & \text{if } \tilde{B} \in \Omega^2_T \doteq \{\tilde{B} : c_T \mathbf{1}'\mathbf{K}^\mathbf{1}_\mathbf{T} - E_T \leq \tilde{B} < c_T \mathbf{1}'\mathbf{K}^\mathbf{1}_\mathbf{T}\} \\ \overline{\overline{\mathbf{K}}}_\mathbf{T} & \text{if } \tilde{B} \in \Omega^3_T \doteq \{\tilde{B} : 0 \leq \tilde{B} < c_T \mathbf{1}'\mathbf{K}^\mathbf{1}_\mathbf{T} - E_T\} \end{cases} \tag{3.1}$$

$$e^*_T(\tilde{B}) = \left(c_T \mathbf{1}'\mathbf{K}^*_\mathbf{T}(\tilde{B}) - \tilde{B}\right)^+. \tag{3.2}$$

The explicit expressions for the capacity vectors in the proposition are given in the proof. $\mathbf{K}^\mathbf{0}_\mathbf{T}$ is the optimal capacity investment in the absence of a budget constraint (the "budget-unconstrained optimal capacity"). If the budget realization is high enough to cover the corresponding cost $c_T \mathbf{1}'\mathbf{K}^\mathbf{0}_\mathbf{T}$ ($\tilde{B} \in \Omega^0_T$), then $\mathbf{K}^*_\mathbf{T}(\tilde{B}) = \mathbf{K}^\mathbf{0}_\mathbf{T}$ with no borrowing. Otherwise, for each budget level $\tilde{B} \in \Omega^{123}_T$, the firm determines to borrow or not by comparing the marginal revenue from investing in an additional unit of capacity over its available budget with the marginal cost of that investment including the external financing cost, $(1+a_T)c_T$. For $\tilde{B} \in \Omega^1_T$, the budget is insufficient to cover $\mathbf{K}^\mathbf{0}_\mathbf{T}$, and the marginal revenue of capacity is lower than its marginal cost. Therefore, the firm optimally does not borrow, and only purchases the capacity level $\overline{\mathbf{K}}_\mathbf{T}$ that fully utilizes its budget $\tilde{B}$. For $\tilde{B} \in \Omega^{23}_T$, the marginal revenue of capacity is higher than its marginal cost $(1 + a_T)c_T$. Therefore, the firm optimally borrows from external markets to invest in capacity. $\mathbf{K}_\mathbf{T}^\mathbf{1}$ is the optimal capacity investment with borrowing, in the absence of a credit limit (the "credit-unconstrained optimal capacity"). If the budget realization and the credit limit can jointly cover its cost, $\mathbf{K}_\mathbf{T}^\mathbf{1}$ is the optimal capacity investment; otherwise, the firm purchases the capacity level $\overline{\overline{\mathbf{K}}}_\mathbf{T}$ that fully utilizes its budget and its credit limit.

The optimal external borrowing level $e_T^*(\tilde{B})$ is such that the firm borrows exactly what it needs to cover its capacity investment. Since production is costless, the firm does not incur any further costs beyond this stage. The firm only borrows for funding the capacity investment, which yields (3.2).

The optimal expected (stage 1) equity value of the firm with a given budget level $\tilde{B}$, $\pi_T(\tilde{B})$, can be obtained in closed form.

Corollary 7 *$\pi_T(\tilde{B})$ is concave increasing in $\tilde{B}$ on* $[0, \infty)$.

3.4.3 Stage 0: Financial Risk Management Level and Technology Choice

In this stage, the firm decides on the technology choice $T \in \{D, F\}$, and the financial risk management level H_T, the number of forward contracts written on the stage 1 asset price α_1. The optimal expected (stage 0) equity value $\Pi^*(\mathbf{W})$ as a function of the internal (stage 1) endowment $\mathbf{W}' = (\omega_0, \omega_1)$ is

$$\Pi^*(\mathbf{W}) = \max\{Z, \omega_0 + \overline{\alpha}_1\omega_1 + P\}. \tag{3.3}$$

Here, Z denotes the expected (stage 0) equity value of the better technology calculated at the optimal risk management level H_T^*. In (3.3), the firm compares this equity value with $\omega_0 + \overline{\alpha}_1\omega_1 + P$, the expected (stage 0) equity value of not investing in any technology. §3.4.3 derives H_T^* and §3.4.3 characterizes the optimal technology choice. This characterization is valid for any continuous α_1 and $\boldsymbol{\xi}$ distribution with positive support and bounded expectation.

Financial Risk Management

The expected direct gain from the financial contract is 0 due to the fair pricing assumption. At the same time, financial risk management affects the distribution of the stage 1 budget $B(\alpha_1, H_T)$, which is used to finance the firm's capacity investment. In choosing H_T, the goal of the firm is to engineer its budget to maximize the expected gain from the technology commitment made in stage 0.

Proposition 13 *It is optimal for the firm to fully hedge:* $H_T^* = \omega_1$.

Full hedging is optimal because π_T is a concave function of the available budget $\tilde{B}$. This follows by Jensen's inequality: For concave π_T, $\mathbf{E}\left[\pi_T(B(\alpha_1, H_T))\right] \leq \pi_T(E[B(\alpha_1, H_T]) = \pi_T(\omega_0 + \overline{\alpha}_1\omega_1)$, the equity value under full hedging.

Technology Choice

We now turn to the technology selection problem. Since the credit limit E_T is uniquely determined by the unit financing cost a_T ($E_T = \frac{P}{1+a_T}$), we only use a_T to denote the loan contract terms. We define the vectors $\mathbf{a} \doteq (a_D, a_F)$ and $\mathbf{H}^* \doteq (H_D^*, H_F^*)$. The choice T^* between flexible versus dedicated technology is determined by a unit cost threshold that makes the firms indifferent between the two technologies.

Proposition 14 *For a given financing cost scheme a_T, and under the financial risk management level $H_T^* = \omega_1$ for each technology $T \in \{D, F\}$, there exists a unique variable cost threshold $\overline{c}_F(c_D, \mathbf{a}, \mathbf{H}^*)$ such that when $c_F < \overline{c}_F(c_D, \mathbf{a}, \mathbf{H}^*)$ it is more profitable to invest in flexible technology ($T^* = F$). Without financial risk management, there is a parallel threshold $\overline{c}_F(c_D, \mathbf{a}, \mathbf{0})$. With symmetric financing costs $a_F = a_D$,*

$$\overline{c}_F(c_D, \mathbf{a}, \mathbf{H}^*) = \overline{c}_F(c_D, \mathbf{a}, \mathbf{0}) = \overline{c}_F^S(c_D) = c_D \left(\frac{\mathbf{E}^{-b}\left[\left(\xi_1^{-b} + \xi_2^{-b}\right)^{-\frac{1}{b}}\right]}{2\overline{\xi}^{-b}} \right)^{-\frac{1}{b+1}} \geq c_D. \quad (3.4)$$

Investing in T^* dominates not making any technology investment.

The cost threshold developed in Proposition 14 reveals the more profitable technology. Proposition 14 also concludes that investing in this technology is more profitable than not investing at all. This completes the characterization of the firm's optimal decisions. The next section analyzes the creditor's problem.

3.5 Analysis of the Creditor's Problem

As the Stackelberg leader, the creditor determines credit terms to ensure an expected return of U: $\mathbf{E}\left[\Lambda_T(a_T, E_T)\right] = U$. Recall from Assumption 15 that at a given unit cost a_T, the creditor offers the credit limit that can be secured with the collateral value P, $E_T(a_T) = \frac{P}{1+a_T}$. Then we can write the stage 0 expected return of the creditor with a loan commitment contract $(a_T, E_T(a_T))$ as

$$\mathbf{E}\left[\Lambda_T(a_T)\right] = a_T\,\mathbf{E}\left[e_T\right] - BC\,\mathbf{E}\left[Pr\left\{\Gamma_T < e_T(1+a_T)\right\}\right], \tag{3.5}$$

where BC is the fixed cost of bankruptcy and Γ_T is the firm's optimal stage 2 operating profits under technology choice T. Since we focus on fully-secured loan commitment contracts ($e_T(1+a_T) \leq P$), the creditor always retrieves the face value of the debt and generates expected earnings of $a_T\mathbf{E}[e_T]$. However, since the physical assets are illiquid, default can occur because the firm is not able to immediately pay back the debt with the liquid assets, in which case the creditor incurs the bankruptcy cost BC.

By Proposition 13, the firm always fully hedges, so, we need only focus on the budget level $\overline{B}$ to expand (3.5). The regions in Proposition 12 depend on a_T. If $\overline{B} \in \Omega_T^{01}(a_T)$, the firm does not borrow. It can also be shown that under our assumptions, the firm never borrows at the credit limit $E_T(a_T)$ for any a_T. Therefore, the firm only borrows from the creditor if $\overline{B} \in \Omega_T^2(a_T)$, where $e_T = c_T\mathbf{1'K_T^{\frac{1}{b}}} - \overline{B} < E_T$. For any a_T satisfying $\overline{B} \in \Omega_T^2$, we have

$$\mathbf{E}\left[\Lambda_T(a_T)\right] = (c_T\mathbf{1'K_T^{\frac{1}{b}}} - \overline{B})a_T - BC\ Pr\left(\mathbf{N'_T1'K_T^{\frac{1}{b}}}^{1+\frac{1}{b}} < (c_T\mathbf{1'K_T^{\frac{1}{b}}} - \overline{B})(1+a_T)\right). \tag{3.6}$$

Let $EE_T \doteq (c_T\mathbf{1'K_T^{\frac{1}{b}}} - \overline{B})a_T$ denote the expected earnings (without default) of the creditor, $P_T \doteq Pr\left(\mathbf{N'_T1'K_T^{\frac{1}{b}}}^{1+\frac{1}{b}} < (c_T\mathbf{1'K_T^{\frac{1}{b}}} - \overline{B})(1+a_T)\right)$ denote the default probability and $ED_T = BC\ P_T$ denote the expected default cost of the creditor. To build intuition into the creditor's problem, we analyze each of the two parts separately and demonstrate the effect of different parameters on each part. For the comparative statics analysis, we focus on local results where $\overline{B} \in \Omega_T^2$ holds.

Proposition 15 *EE_T decreases in the unit cost of the technology c_T and firm size $\overline{B}$. There exists a unique threshold $\underline{a}_T$ such that EE_T increases in a_T for $a_T < \underline{a}_T$ and decreases in a_T for $a_T > \underline{a}_T$.*

A larger unit investment cost induces the firm to decrease its capacity investment level. As a result, the total investment cost decreases, and so does the borrowing level. As the budget of the firm increases, it's obvious that its borrowing level would decrease. As the financing cost a_T increases, the optimal investment level, and hence the borrowing level decreases. For small levels of a_T, the increase in the marginal return of increasing a_T dominates the reduction in the borrowing level. As a_T becomes larger the percentage reduction in the borrowing level becomes more significant and dominates the positive effect of increasing marginal returns. Hence the result in Proposition 15 follows.

Proposition 16 *Let P_T denote the expected default probability with technology $T \in \{F, D\}$. For $\overline{B} \in \Omega_T^2$,*

$$P_T = Pr\left(H_T(\boldsymbol{\xi}) < M_T(1+\frac{1}{b})\left[1 - \frac{\overline{B}}{c_T \mathbf{1}'\mathbf{K}_\mathbf{T}^\mathbf{1}}\right]\right) \tag{3.7}$$

where $\overline{B} = \omega_0 + \overline{\alpha}_1\omega_1$, $M_F = \mathbf{E}\left[\left(\xi_1^{-b} + \xi_2^{-b}\right)^{-\frac{1}{b}}\right]$, $M_D = 2^{-1/b}\overline{\xi}$, $H_F(\boldsymbol{\xi}) = \left(\xi_1^{-b} + \xi_2^{-b}\right)^{-\frac{1}{b}}$, and $H_D(\boldsymbol{\xi}) = \frac{\xi_1+\xi_2}{2^{1+1/b}}$. P_T decreases in firm size (expected budget level $\overline{B}$) and in unit investment cost c_T for an arbitrary level of a_T. P_T decreases in unit financing cost a_T.

For an arbitrary level of a_T, an increase λ reduces the borrowing level while not affecting the other factors, so the default probability decreases. Hence large firms have lower default risk than small firms at the same level of a_T. One may expect a higher technology cost to result in a higher borrowing level and hence a higher default risk, but the firm's optimal capacity investment level decreases in c_T, leading to a decrease in the borrowing level. Therefore an increase in c_T leads to a lower default risk at a given level of a_T. As the unit financing cost a_T increases, the capacity investment level decreases. Both the operating cash flows decrease and the face value of the loan decrease, but the net effect is that the default probability decreases.

It is interesting to note that the default probability decreases in the unit financing cost. Melnik and Plaut (1986) derive several relations among the parameters of loan commitment contract after assuming that the borrowing level is independent of the unit financing cost, and that the default probability increases in the unit financing cost. Propositions 15 and 16 demonstrate that these assumptions may not be valid with a more formal representation of operations.

We have characterized the expected return of the creditor for a given a_T; we now characterize equilibrium financing cost a_T^* for each technology. We focus on the the Pareto-Nash equilibrium that achieves a return of U for the creditor and the highest profit for the firm. This is consistent with the financial economics literature where the creditor makes zero expected return in equilibrium ($U = 0$) and the creditor chooses the best contract for the firm. In our model, the Pareto-optimality refinement guarantees the uniqueness of the equilibrium financing cost a_T^*, if such a cost exists:

Proposition 17 *If there exists a feasible $a_T \geq 0$ that satisfies $\mathbf{E}[\Lambda_T(a_T)] = U$ for the creditor, then in the Pareto-optimal equilibrium, the creditor offers a unique loan commitment contract for each technology T with parameters*

$$\begin{aligned} a_T^* &= argmin_{a_T \geq 0}\, \mathbf{E}[\Lambda_T(a_T)] = U, \\ E_T^* &= \frac{P}{1+a_T^*}. \end{aligned}$$

If such an $a_T \geq 0$ that satisfies $\mathbf{E}[\Lambda_T(a_T)] = U$ does not exist, then in equilibrium the creditor does not offer a contract. In this case we say $a_T^* \to \infty$ and $E_T^* \to 0$.

The minimal a_T (which also corresponds to maximal credit limit E_T) is Pareto-optimal for the firm because the expected equity value of the firm (weakly) increases as more external capital becomes available at a lower unit cost; the creditor is indifferent between all a_T's satisfying his reservation expected utility U. Since the firm's borrowing level depends on a_T, when the fixed bankruptcy cost or the reservation utility of the creditor is sufficiently large, then there may not exist a feasible a_T that satisfies the creditor's requirement of U. In this case, the creditor does not offer a contract and the firm cannot raise external capital.

Proposition 18 *The equilibrium level of unit financing cost a_T^* and credit limit E_T^* increase and decrease, respectively, in the fixed cost of bankruptcy and the underwriter fee. The optimal (stage 0) expected equity value of the firm and the expected capacity investment level decrease in the fixed cost of bankruptcy and the underwriter fee.*

It is intuitive that higher bankruptcy costs and underwriter fees induce the creditor to ask for a higher level of unit financing cost (which results in a lower credit limit). Higher imperfection costs in the financial markets are directly transferred to the firm's operations, resulting in a lower expected equity value.

3.6 The Perfect Capital Market Benchmark

The goal of this paper is to understand the effect of capital market imperfections on the firm's operational decisions and performance. As mentioned in the introduction, the capacity investment literature has implicitly assumed frictionless borrowing, from which follows a series of results on optimal capacity investment and technology choice. We will show that many of these conclusions do not hold once capital market imperfections are taken into account. To this end, we first identify the natural perfect market benchmark in our modeling framework.

Proposition 19 *In our model, when capital markets are perfect, i.e. when there are no bankruptcy and underwriter fees, the equilibrium unit financing cost is the risk-free rate ($a_T^* = 0$) and the equilibrium credit limit is the value of the collateralized physical assets of the firm ($E_T^* = P$).*

The perfect market assumption prohibits any transaction costs (e.g. fixed cost of bankruptcy, underwriter fee) and requires the fair valuation of the debt obligation in terms of its underlying default exposure. Since we focus on collateralized debt, in the absence of transaction costs ($BC = 0, U = 0$), there is no risk for the creditor associated with default. Consequently, the fair price of any secured debt obligation is the risk-free rate ($a_T^* = 0$)[2] and the credit limit E_T is the value of the collateralized physical asset ($E_T^* = P$). If there are capital market imperfections, then $a_T^* > 0$ and $E_T^* < P$ in our framework.

Proposition 20 *If the capital markets are perfect, the firm's operational decisions are independent of the financing and financial risk management decisions. Financial risk management does not have any value. The firm invests in the budget-unconstrained capacity investment level for any budget realization,* $\mathbf{K_T^*}(\tilde{B}) = \mathbf{K_T^0}$ *and borrows to finance this capacity level,* $e_T^*(\tilde{B}) = [c_T \mathbf{1}'\mathbf{K_T^0} - B]^+$*. Equilibrium technology choice* T^* *is determined by the variable cost threshold* $\bar{c}_F^P(c_D) = \bar{c}_F^S(c_D)$ *of Proposition 14 and the expected (stage 0) equity value in equilibrium is given by* $\Pi^*(\mathbf{W}) = \overline{B} + \frac{c_T \mathbf{1}'\mathbf{K_T^0}}{-(b+1)} + P$.

The optimal investment level $\mathbf{K_T^0}$ is identical to the one in traditional stochastic capacity models (Van Mieghem 2003). The firm simply chooses the optimal investment level without regard to the budget limit or financing costs, and implements it by borrowing if necessary. This is consistent with the decoupling of operational and financial decisions in perfect markets (Modigliani and Miller 1958). In the same line of reasoning, financial risk management has no value in perfect capital markets (Fite and Pfleiderer 1995) with which Proposition 20 is again consistent.

[2]If the debt were not fully secured, then the unit cost of financing would be larger than the risk-free rate even in a perfect market (see Xu and Birge 2004 for a detailed treatment of this with unsecured spot lending).

3.7 Effect of Capital Market Imperfections on the Firm's Operational Decisions and Performance - The Single Product Case

In this section, we characterize the creditor-firm equilibrium for specific demand distributions and a single product. We compare the results with the perfect market benchmark of Section 3.6 to demonstrate how conclusions arrived at under the (implicit) perfect market assumption may change when imperfections are taken into account. Our goal is not to undertake a complete characterization of the equilibrium, but to show the existence of certain heretofore unidentified effects. We focus on the firms that engages in financial risk management (except the last part where we analyze the value of financial risk management). All the analytical results of this section continue to hold for the firms that do not use financial risk management with minor modifications in the proofs.

In the single product setting, the firm uses a single resource and technology choice is not relevant so we eliminate the D and F subsripts. The optimal capacity investment, production quantity and financial risk management decisions of the firm follow from our analysis in §3.4 by setting the range of one of the product market uncertainties ξ_i to 0; we also eliminate the i subscript. Unless otherwise specified, this section assumes that ξ is uniformly distributed between $[0, 2\bar{\xi}]$. With this assumption, the expected return of the creditor for a given a satisfying $\overline{B} \in \Omega_2(a)$ can be written as

$$\mathbf{E}\left[\Lambda(a)\right] = \left(cK^0(1+a)^b - \overline{B}\right)\left(a - \frac{BC\;\bar{\xi}(1+1/b)}{2\bar{\xi}cK^0(1+a)^b}\right).$$

The uniform distribution assumption leads to a nice decomposition of the creditor's expected return in product form: The first term is the expected amount of lending. The second term is the expected unit marginal profit of lending. For each unit of the loan, the creditor earns a and incurs an expected default cost. This decomposition holds because the default risk is a linear function of the expected amount of lending under the uniform distribution. The following proposition characterizes the equilibrium unit financing cost:

Proposition 21 *If the product market uncertainty ξ is uniformly distributed in* $[0, 2\overline{\xi}]$ *and $U \geq 0$, there exists a unique bankruptcy cost threshold $\widehat{BC}$ such that for $BC > \widehat{BC}$, the firm does not lend money in equilibrium. For $BC \leq \widehat{BC}$, if U and $\overline{B}$ are sufficiently small, the creditor offers a rate $a^* < \left(\frac{cK^0}{\overline{B}}\right)\left(\frac{cK^0}{\overline{B}}\right)^{-1/b}$ and the firm borrows; otherwise the creditor does not offer a contract or offers the rate $a^* = \left(\frac{cK^0}{\overline{B}}\right)^{-1/b} - 1$ and the firm does not borrow.*

If the bankruptcy cost is sufficiently high, then the expected marginal profit is negative for the creditor for all feasible a. In this case, the the creditor does not offer any loan. For a sufficiently low level of bankruptcy cost, the expected marginal profit can be positive for some range of a. In this case, if U and $\overline{B}$ are not very high then the creditor offers a finite a^* in equilibrium and the firm borrows. If U is sufficiently high, since the expected loan amount is limited and decreasing in a, the creditor cannot generate sufficient expected returns to satisfy U. If $\overline{B}$ is sufficiently high, since the expected amount of lending is low, similarly, the creditor cannot generate sufficient expected returns to satisfy U. In these cases, loan is not offered in equilibrium. In summary, borrowing only takes place if the bankruptcy cost and underwriter fee are sufficiently low.

We now investigate the effect of product market variability, firm size, unit capacity investment cost and engaging in financial risk management on the equilibrium level of financing costs and firm's decisions and performance. Our focus is to show differences from the perfect market case. For convenience, we summarize the results of the next four propositions in Table 1.

The effect of product market variability. As suggested by traditional models, and as also follows from Proposition 20, in perfect capital markets, the firm's capacity decision and the expected (stage 0) equity value depend on the expected value of product market uncertainty but not on product market variability. The following proposition demonstrates that independence from product market variability does not hold in imperfect capital markets.

Proposition 22 *If the product market uncertainty ξ is uniformly distributed in* $[0, 2\overline{\xi}]$,

	Perfect Market	Imperfect Market
Product market variability	Does not impact capacity level and equity value	Decreases capacity level and equity value
Expected budget level	Does not impact capacity level and increases equity value	May decrease capacity level and equity value
Unit investment cost	Decreases the capacity level and equity value	Decreases the capacity level and equity value even more
Financial risk management	Does not have any value	Has positive static and possibly negative strategic value

Table 3.1: Differences between perfect and imperfect markets in single-product investments with uniform $[0, 2\bar{\xi}]$ product market uncertainty

the expected optimal capacity investment level and (stage 0) equity value of the firm in equilibrium decrease in product market variability through an increase in the equilibrium financing cost level.

This result also holds when the product market uncertainty ξ is normal or uniform with mean $\bar{\xi}$. For an arbitrary financing cost a, increasing variability alters neither the capacity investment level nor the expected (stage 0) equity value of the firm. The firm is only concerned with the mean of the product market uncertainty; this is in line with the perfect market benchmark. With normal and uniform distributions, higher variability corresponds to the mean-preserving spread of ξ – more probability mass is transferred to the tails. Since the creditor is only concerned with the downside risk of the firm's operating cash flows; higher variability translates into higher downside risk. This leads to higher expected default risk for the firm and lower expected returns for the creditor. To compensate for this reduction, the creditor charges higher financing cost in equilibrium. In turn, the increase in unit financing cost decreases the capacity investment level and the operational performance of the firm in equilibrium.Proposition 22 provides an analytical proof for the numerical observation of Xu and Birge (2004) on the effect of product market variability in a single-product setting.

The effect of firm size. We proxy the firm size with the expected internal (stage 1) endowment ($\overline{B}$) of the firm. In perfect capital markets, an increase in the expected internal endowment does not alter the optimal capacity investment level, but increases the expected equity value due to the increase in the term $\overline{B}$. In imperfect capital markets, a change in $\overline{B}$ also has an indirect effect through altering the equilibrium level of financing cost.

Proposition 23 *If the product market uncertainty ξ is uniformly distributed in $[0, 2\overline{\xi}]$, the equilibrium unit financing cost increases in the expected budget level. The expected optimal capacity investment level decreases in the expected budget level if the firm borrows in equilibrium and increases otherwise. The expected (stage 0) equity value of the firm may decrease or increase in the expected budget level.*

At a given a, the expected marginal profit of the creditor is independent of the internal endowment of the firm; but a higher endowment level means a smaller loan as $\overline{B}$ is larger. As a result, the creditor increases the unit financing cost in equilibrium to compensate for this reduction. Since the expected optimal capacity investment level depends on $\overline{B}$ only when the firm does not borrow, increasing a^* decreases the capacity investment level when the borrowing takes place. The combined effect of increasing $\overline{B}$ and increasing a^* on the expected (stage 0) equity value can be positive or negative.

In the literature, it has been argued qualitatively that larger firms can obtain lower financing costs than smaller firms, based on the premise that larger firms have more internal capital and borrow less; and hence have lower default risk. Proposition 23 highlights the importance of the expected borrowing level, and demonstrates that the lower external borrowing need of large firms may induce the creditor to charge a higher unit financing cost in equilibrium. Proposition 23 also demonstrates that larger firms may not perform better than smaller firms despite larger firms' higher internal endowment.

The effect of unit capacity investment cost. In perfect capital markets, an increase in the unit capacity investment cost decreases the expected equity value through reducing the optimal capacity investment level. In imperfect capital markets, an increase in c reduces these quantities even more due to an increase in the external financing cost.

Proposition 24 *If the product market uncertainty ξ is uniformly distributed in $[0, 2\bar{\xi}]$, the equilibrium level of financing cost increases, the expected optimal capacity investment level and (stage 0) equity value of the firm decrease in the unit capacity investment cost.*

In §3.5, we demonstrated that in imperfect capital markets, an increase in c_D decreases the expected earnings but also decreases the expected bankruptcy cost of the creditor. The combined effect is indeterminate and depends on the product market uncertainty. With a uniform distribution, the first effect dominates the second effect. Therefore, in equilibrium, the creditor increases the unit financing cost to compensate for the reduction in expected returns.

The effect of engaging in financial risk management. As we demonstrated in Proposition 20, financial risk management does not have any value for the firm in perfect capital markets. It follows from Proposition 19 that the firm engaging in financial risk management does not have any effect on the creditor's returns in perfect markets. Under market imperfections, the financial risk management policy of the firm affects both the firm and the creditor. We first focus on the value of engaging in financial risk management for the firm. Let Π^{FRM} and Π^{-FRM} denote the expected (stage 0) equity value of the firm with and without engaging in financial risk management respectively. The value of engaging in financial risk management for the firm, Δ, is given by

$$\begin{aligned} \Delta &= \Pi^{FRM}(a^*_{FRM}) - \Pi^{-FRM}(a^*_{-FRM}) \\ &= \left[\Pi^{FRM}(a^*_{FRM}) - \Pi^{FRM}(a^*_{-FRM})\right] + \left[\Pi^{FRM}(a^*_{-FRM}) - \Pi^{-FRM}(a^*_{-FRM})\right]. \end{aligned}$$

The second term in brackets is the *static* value of engaging in financial risk management for the firm. At a given unit financing cost a^*_{-FRM}, the firm uses forward contracts to engineer its internal cash flow to avoid dependence on external borrowing. The static value is always non-negative because the firm can always choose not to engage in financial risk management.

The first term in brackets corresponds to the *strategic* value of engaging in financial risk management for the firm, obtained by altering the equilibrium level of financing cost by changing the expected returns of the creditor. The strategic value is negative if engaging in financial risk management increases the equilibrium level of unit financing cost; it is positive otherwise. For an arbitrary a, engaging in financial risk management decreases the expected earnings of the creditor, because the firm borrows less in expectation with financial risk management. The effect on the expected default cost is mainly determined by the product market uncertainty. The following proposition characterizes the equilibrium unit financing cost and the strategic value of financial risk management for the uniform case.

Proposition 25 *Let a^*_{FRM} (a^*_{-FRM}) denote the equilibrium financing cost with (without) financial risk management. Let the product market uncertainty ξ be uniformly distributed in $[0, 2\bar{\xi}]$. For $BC > \widehat{BC}$, the creditor does not offer financing with or without financial risk management. For $BC \leq \widehat{BC}$,*

1. *If $U = 0$, then $a^*_{FRM} \leq a^*_{-FRM}$ but there is no strategic value.*
2. *If $U > 0$ and $\overline{B}$ are sufficiently small such that a finite a^*_{FRM} exists, then $a^*_{FRM} > a^*_{-FRM}$ and financial risk management has negative strategic value.*
3. *If $U > 0$ is sufficiently large and $\overline{B}$ is sufficiently small such that the creditor would not offer financing to the firm engaging in financial risk management, then either a finite a^*_{-FRM} exists and financial risk management has negative strategic value, or the creditor does not finance the firm without risk management either and financial risk management has no strategic value.*
4. *If $U > 0$ and $\overline{B}$ are sufficiently large such that the creditor would not offer financing to the firm engaging in financial risk management, then either a finite a^*_{-FRM} exists or the creditor does not finance the firm without risk management either and financial risk management has no strategic value.*

It follows from Proposition 25 that engaging in financial risk management may have a negative strategic value for the firm. In the uniform case, since the default risk is a linear function of the borrowing level, the strategic value is determined by the change in expected borrowing level. When the firm borrows in equilibrium without financial risk management, engaging in financial risk management decreases the expected borrowing level, and the creditor increases the unit financing cost to compensate for this reduction. This is in contrast with the documented positive strategic value of financial risk management. Smith and Stulz (1985) demonstrate that when the firm uses financial risk management to hedge the operating cash flows *after the loan is taken*, then engaging in financial risk management decreases the equilibrium level of financing cost by reducing the expected default risk. In our case, financial risk management is effective *before the loan is taken*, and increases the equilibrium level of financing cost by decreasing the expected earnings of the creditor. The total value Δ is determined by the comparison between the positive static value and negative strategic value.

3.8 Effect of Capital Market Imperfections on Firm's Decisions and Performance – The Two-Product Case

In Section 3.7, we analyzed the effect of capital market imperfections in the single product setting in which the technology choice of the firm is irrelevant. In this section, we focus on the effect of capital market imperfections in the two-product setting where technology choice is a non-trivial question and demand correlation matters. In §3.8.1, we investigate the effect of product market uncertainty (demand correlation ρ and demand variability σ) on the optimal capacity investment level and the operational performance of the firm with each technology. In §3.8.2, we analyze the impact of capital market imperfections on the firm's technology choice. We use the perfect market benchmark that we developed in §3.6 to delineate the effect of capital market imperfections and highlight the new trade-offs that arise. Our main results are summarized in Table 3.2.

In this section, to carry out comparative statics analysis with respect to ρ and σ, we assume that $\boldsymbol{\xi}$ has a bivariate normal distribution. We use the same parameter set ($c_F = c_D = 2, P = 220, b = -2, \overline{\xi}_1 = \overline{\xi}_2 = 20, \overline{B} = 5, U = 1$) for the numerical examples throughout this section. As in Section 3.7, we focus on the firms that engage in financial risk management throughout this section. All the analytical results continue to hold for the firms that do not use financial risk management.

		Perfect Market	Imperfect Market
Dedicated Technology	Increase in variability	Does not impact capacity level and equity value	Decreases capacity level and equity value
	Increase in correlation	Does not impact capacity level and equity value	Decreases capacity level and equity value
Flexible Technology	Increase in variability	Increases capacity level and equity value	May decrease capacity level and equity value
	Increase in correlation	Decreases capacity level and equity value	May increase capacity level and equity value
Risk Pooling Value ($c_F = c_D$)	Increase in variability	Increases risk pooling value	May decrease risk pooling value
	Increase in correlation	Decreases risk pooling value	May decrease or increase risk pooling value
Technology choice	$c_F = c_D$	Flexible is always preferred	Dedicated may be preferred to flexible

Table 3.2: Differences between perfect and imperfect markets in two-product investments with bivariate normal product market uncertainty

3.8.1 The Effect of Capital Market Imperfections for a Given Technology

Dedicated Technology. Similar to the single product case, in perfect capital markets, the firm's capacity decision and the expected (stage 0) equity value with the dedicated technology depend only on the mean demand vector $\bar{\boldsymbol{\xi}}$ and not on the covariance matrix $\boldsymbol{\Sigma}$. In imperfect capital markets, if the firm borrows in equilibrium elements of the covariance matrix (ρ and σ) also matters.

Proposition 26 *If the product market uncertainty $\boldsymbol{\xi}$ has symmetric bivariate normal distribution, then the optimal expected capacity investment level and (stage 0) equity value of the firm using dedicated technology decrease in the demand correlation ρ and the demand variability σ through an increase in the equilibrium level of unit financing cost.*

As in the single product case, higher variability increases the default risk of the firm, which reduces the expected return of the creditor for an arbitrary financing cost a_D. The creditor increases the equilibrium level of financing cost to compensate for this reduction.

The result with respect to demand correlation follows from a financial risk-pooling argument. The firm's default probability for a given capacity choice depends on the variability in operating revenues. Operating in two markets creates a *diversification benefit* for the firm: When the product markets are negatively correlated, the firm generates high returns from one market and low returns from the other, reducing revenue variability and hence default risk. With high positive correlation, the firm generates similar revenues from both markets, increasing its default risk. Therefore, as correlation increases, the creditor increases the equilibrium level of financing cost to compensate for the increase in expected bankruptcy costs. Increase in the equilibrium level of financing cost leads to a reduction in the total capacity investment level and the expected (stage 0) equity value of the firm as Figure 3.1 demonstrates.

The financial risk-pooling effect discussed in this section is different from the risk-pooling effect of flexible technology that comes from the ability to switch capacity between products. The former effect only exists in imperfect capital markets, whereas the latter is a product-market effect and also exists in perfect capital markets.

Flexible Technology. In perfect capital markets, the firm's capacity investment decision and its expected (stage 0) equity value with flexible technology do depend on

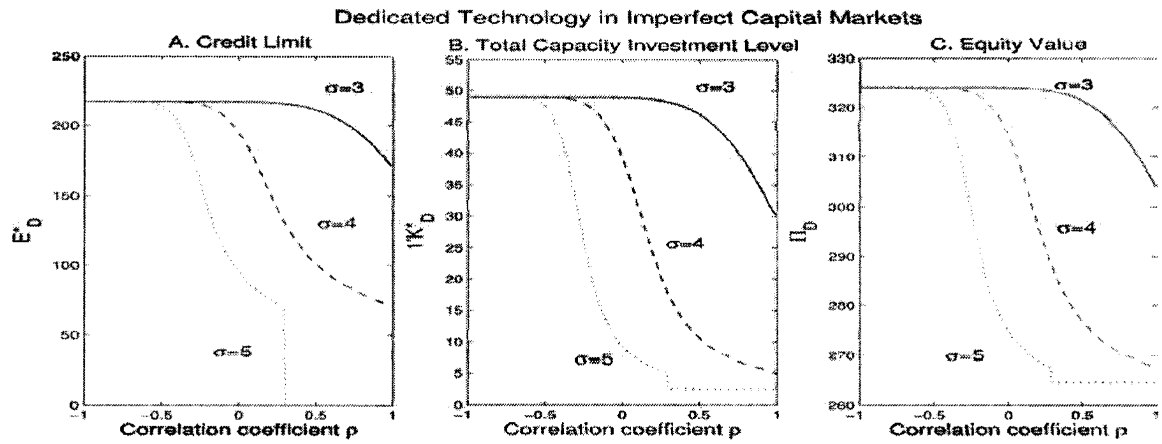

Figure 3.1: Effect of demand correlation ρ and demand variability σ on the dedicated technology investment in imperfect markets: Higher ρ and σ leads to lower credit limit E_D^* in equilibrium (Panel A) and this decreases the total capacity investment level (Panel B) and the expected (stage 0) equity value of the firm (Panel C).

the covariance matrix $\mathbf{\Sigma}$ of $\boldsymbol{\xi}$ through the term $M_F \doteq E\left[\left(\xi_1^{-b} + \xi_2^{-b}\right)^{-\frac{1}{b}}\right]$ (Proposition 20). This term captures the risk-pooling value of flexible technology that comes from the ability to switch production between two products after demand uncertainty is resolved. Under general correlation and variability measures, M_F decreases with increasing ρ and decreasing σ (Boyabatlı and Toktay 2006a). Therefore, the optimal expected capacity investment level and (stage 0) equity value of the firm decreases with increasing ρ and decreasing σ in perfect capital markets. We call this the *direct (static) effect* of ρ and σ.

In imperfect capital markets, demand correlation ρ and the demand variability σ also have an *indirect (strategic) effect* as they alter the equilibrium level of financing cost. Two different drivers give rise to the indirect effect of ρ and σ: the expected value of production switching for a given capacity investment level and the optimal capacity investment level that incorporates this switching value. These two drivers have different effects on the equilibrium level of financing cost.

As the value of the switching option at a given capacity level decreases, the expected default probability of the firm increases. An increase in ρ or a decrease in σ typically decreases the value of production switching and increases the firm's default probability for a given capacity investment level[3]. This acts to increase the equilibrium financing cost.

Anticipating the expected production switching value, the firm optimally adjusts its capacity investment level to exploit the benefits of production switching. With an increase in ρ and a decrease in σ, the firm optimally invests less in capacity. A lower capacity investment level means lower expected earnings for the creditor but also a lower expected default cost. Lower expected earnings (without default) act to increase the equilibrium financing cost, while a lower expected default cost acts to decrease it. The net effect is indeterminate.

To summarize, the overall effect of demand correlation ρ and demand variability σ on the optimal capacity investment level and the expected (stage 0) equity value of the firm with flexible technology is indeterminate in imperfect capital markets. This effect is a combination of the indirect effect, where the two major drivers may work in opposite directions, and the direct effect, which is the same as in the perfect market case. Figure 3.2 demonstrates that any effect can dominate: Different from perfect market setting, the (stage 0) equity value of flexible technology may increase with an *increase* in demand correlation and a *decrease* in demand variability.

3.8.2 The Effect of Capital Market Imperfections on Technology Choice

For each technology cost pair (c_F, c_D), there exists a unique unit cost threshold $\bar{c}_F(c_D, a_F^*(c_F), a_D^*(c_D))$[4] that determines the optimal technology choice of the firm (Propositions 14 and 17). The optimal technology choice captured by this threshold

[3]It can be shown that the default probability increases for ξ that has a bivariate normal distribution when we go from $\rho = -1$ to $\rho = 1$ and $\sigma > 0$ to $\sigma = 0$.

[4]We drop the argument $\mathbf{H}^*$ in the cost threshold, because the firm optimally fully hedges for any c_T, a_T for technology $T \in \{D, F\}$.

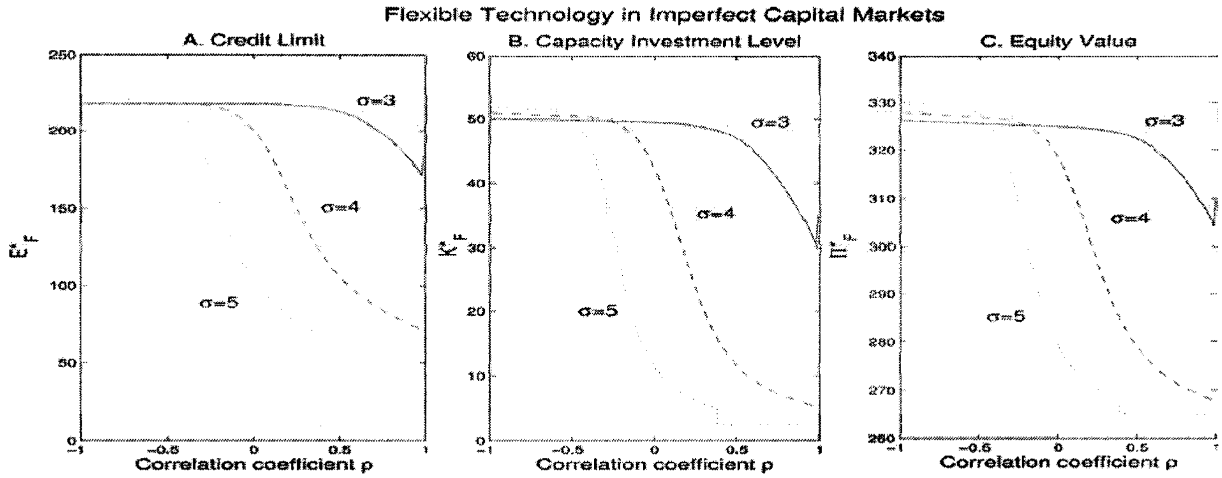

Figure 3.2: Effect of demand correlation ρ and demand variability σ on the flexible technology investment in imperfect markets: In equilibrium, higher σ leads to lower credit limit E_F^* (Panel A), higher ρ leads to lower credit limit E_F^* for low correlations (Panel A) and may lead to higher credit limit E_F^* for close to perfect positive correlations (Panel A, $\sigma = 3$). Sufficiently large decrease in E_F^* may lead to lower capacity investment (Panel B) and expected (stage 0) equity value (Panel C) for increasing σ. For close to perfect positive correlations, higher ρ may lead to higher capacity investment (Panel B, $\sigma = 3$) and expected (stage 0) equity value (Panel C, $\sigma = 3$).

is based on comparing the higher investment cost of flexible technology against its (potential) production switching value relative to dedicated technology. As we shall see, putting these two effects together, there are cases where flexible technology is preferred in perfect markets, but the firm chooses dedicated technology in imperfect capital markets. In this section, we explain the main driver for this result: the strategic effects of investment cost and production switching capability on financing costs in equilibrium.

We start with analyzing the production switching value of flexible technology under identical unit capacity investment costs ($c_F = c_D$), defined as the stage 0 equity value difference between the two technology choices:

$$\pi_F(K_F^*; a_F^*(K_F^*)) - \pi_D(\mathbf{K_D^*}; a_D^*(\mathbf{K_D^*})) = \tag{3.8}$$
$$\underbrace{[\pi_F(K_F^*; a_D^*(\mathbf{K_D^*})) - \pi_D(\mathbf{K_D^*}; a_D^*(\mathbf{K_D^*}))]}_{\text{Static Value}} + \underbrace{[\pi_F(K_F^*; a_F^*(K_F^*)) - \pi_F(K_F^*; a_D^*(\mathbf{K_D^*}))]}_{\text{Strategic Value}}$$

The value of production switching is a combination of two terms, the *static value* and the *strategic value.* The static value is the value of production switching at a given unit financing cost (the equilibrium unit financing cost $a_D^*(\mathbf{K_D^*})$ under dedicated technology in this case). The strategic value captures the effect of production switching on the creditor's expected returns and, hence, the change in the equilibrium level of financing cost with flexible technology.

In perfect markets, the static value of production switching is always positive (as follows from Proposition 20) and the strategic value does not exist (as follows from Proposition 19, we have $a_D^*(\mathbf{K_D^*}) = a_F^*(K_F^*) = 0$ in perfect markets). Production switching enables the firm generate to higher revenues for a given capacity level at stage 2; and the optimal capacity investment decision of the firm increases the expected (stage 0) equity value of the firm even more by optimally exploiting the production switching capability.

In imperfect capital markets, similar to perfect market case, the static value of production switching is always positive. To delineate the strategic value of production switching, we focus on the two fundamental drivers of production switching that we discussed in the previous section: the expected value of production switching for a given capacity investment level and the optimal capacity investment level that incorporates this switching value.

$$\pi_F(K_F^*; a_F^*(K_F^*)) - \pi_F(K_F^*; a_D^*(\mathbf{K_D^*})) = \tag{3.9}$$
$$[\pi_F(K_F^*; a_F^*(\mathbf{K_D^*})) - \pi_F(K_F^*; a_D^*(\mathbf{K_D^*}))] + [\pi_F(K_F^*; a_F^*(K_F^*)) - \pi_F(K_F^*; a_F^*(\mathbf{K_D^*}))]$$

In (3.9), the first term in brackets captures the effect of production switching on the equilibrium level of financing cost at a given capacity investment level ($\mathbf{K_D^*}$) with flexible technology. The second term in brackets demonstrates the effect of the optimal capacity investment decision that incorporates the production switching value, K_F^*, on the equilibrium financing cost.

The first term in brackets in (3.9) is always positive. At a given capacity investment level, the production switching capability of flexible technology decreases the default risk of the firm relative to dedicated technology. This can be observed by noting that for any $\boldsymbol{\xi}$ realization, we have $H_F(\boldsymbol{\xi}) \geq H_D(\boldsymbol{\xi})$ in Proposition 16. Therefore, the expected default cost with flexible technology is lower and all else being equal, the creditor would charge a lower financing cost for the flexible technology: $a_F^*(\mathbf{K_D^*}) \leq a_D^*(\mathbf{K_D^*})$.

The second term in brackets in (3.9) can be positive or negative. Due to the expected production switching value, the firm optimally increases its capacity investment level with flexible technology compared to dedicated technology ($K_F^* \geq \mathbf{1}'\mathbf{K_D^*}$) and optimally borrows more. This induces the creditor to make higher expected returns at $a_F^*(\mathbf{K_D^*})$, and acts to decrease the financing cost. On the other hand, the higher borrowing level increases the default risk of the flexible technology for a fixed $H_T(\boldsymbol{\xi})$. This increases the expected default cost of the creditor and acts to increase the financing cost.

The overall strategic effect of these two value drivers of production switching and consequently, the total production switching value that also incorporates the static value are indeterminate. Panel C of Figure 3.3 demonstrates the strategic value can be negative and dedicated technology can be preferred over flexible technology with identical unit capacity investment cost. Panel D of Figure 3.3 shows that the strategic value of risk pooling may always be positive for any correlation level.

In the limiting case, we can show that the strategic value of production switching does not exist when there is no static value:

Proposition 27 *If the product markets are perfectly positively correlated ($\rho = 1$), for technologies with identical unit capacity investment cost ($c_F = c_D$), the creditor offers the same unit financing cost for both technologies ($a_D^*(\mathbf{K_D^*}) = a_F^*(K_F^*)$), the firm's optimal total capacity investment decision is the same with both technologies ($K_F^* = \mathbf{1}'\mathbf{K_D^*}$), and there is no production switching value of the flexible technology*

$(\pi_F(K_F^*; a_F^*(K_F^*)) = \pi_D(\mathbf{K}_\mathbf{D}^*; a_D^*(\mathbf{K}_\mathbf{D}^*)))$.

With perfect positive correlation, there is no static value for production switching as the traditional models in capacity investment suggest. It follows from Proposition 27 that both terms in (3.9) are zero and there is no strategic value either. Although the

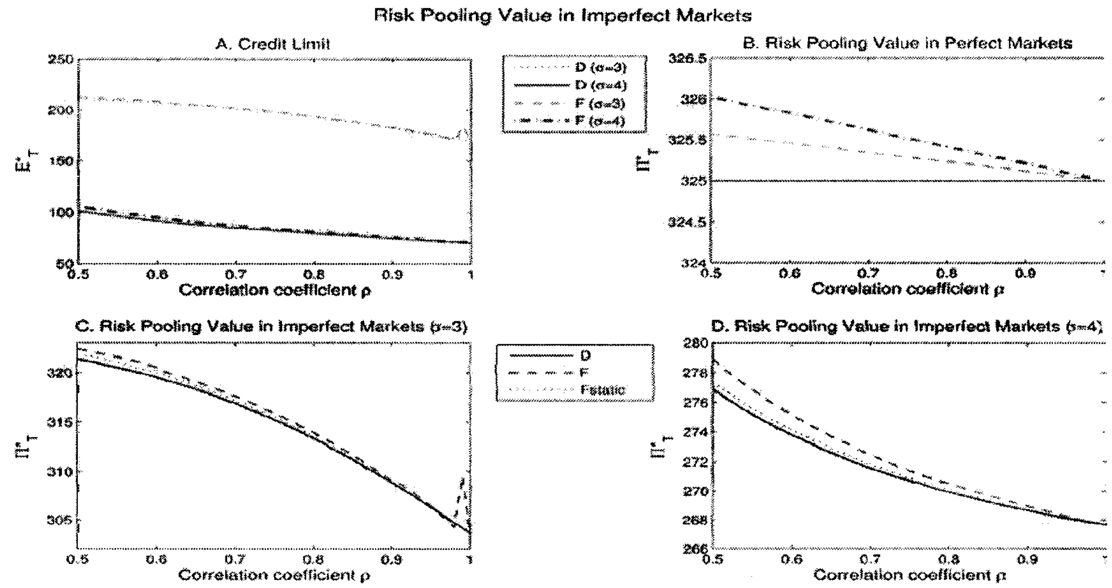

Figure 3.3: Value of Risk Pooling in Imperfect Markets

effects of production switching are best understood when two technology costs are identical ($c_F = c_D$), in general, the firm is concerned with comparing technologies at any given technology cost pair (c_F, c_D). For the technology cost structure $c_F > c_D$, not only the static effect but also the strategic effect of production switching in (3.9) is altered by the higher investment cost of flexible technology. In imperfect capital markets, the sign of the static value of production switching for an arbitrary cost pair (c_F, c_D) coincides with the sign of the static value in perfect capital markets[5]. This property underlines the importance of the strategic value in determining technology choice in imperfect capital markets. As the following proposition demonstrates, the

[5]The sign of the static value is determined by the unit cost threshold $\bar{c}_F^S(c_D)$ as we demonstrated in Proposition 14.

negative strategic value of production switching that exists only in imperfect capital markets can direct the firm towards dedicated technology in these markets.

Proposition 28 *For product market correlation $\rho \approx 1$, and unit capacity investment cost pair $\left(c_F^S(c_D; \rho \approx 1), c_D\right)$, the creditor offers a lower unit financing cost for the dedicated technology ($a_D^* < a_F^*$), and the firm chooses dedicated technology in equilibrium ($T^* = D$).*

The unit cost of flexible technology in this technology cost pair is the threshold derived in Proposition 14: The firm is indifferent between the two technologies in perfect markets. What Proposition 28 shows is that the firm chooses dedicated technology with these same costs if there are capital market imperfections. We explain this as being the result of the negative strategic value of production switching, sufficient to make the total value of the flexible technology negative. To see this, suppose that the creditor offers identical financing costs for each technology. Then the firm borrows the same amount from the creditor, and hence the expected earnings of the creditor (without default) are identical with each technology. However, the default risk with dedicated technology is lower: Production switching is not of high value (because of high correlation) and the firm optimally invests higher capacity with the dedicated technology ($\mathbf{1}'\mathbf{K}_\mathbf{D}^* > K_F^*$). Higher total capacity investment enables the firm to generate sufficient revenues to avoid default with the dedicated technology for some demand realizations in which the firm defaults with the flexible technology. This means that in the Pareto equilibrium that achieves expected return U for the creditor, the financing cost with dedicated technology must be lower. Consequently, the strategic value of production switching is negative, and the firm chooses the dedicated technology in equilibrium.

3.9 Conclusion

This paper contributes to the capacity investment literature by taking capital market imperfections into account and analyzing the interaction of a number of operational and financial decisions in a capacity investment setting. In our model, the firm makes three sequential decisions: technology choice (flexible or dedicated technology), capacity investment and production quantities. The firm's limited internal endowment depends partly on a tradable asset. The firm can borrow from a creditor to finance its operational investments in the capacity investment stage. It can also undertake financial risk management to engineer its internal endowment and reduce its borrowing needs.

The creditor offers two separate secured loan commitment contracts, one for each technology, the terms of which are determined in a Stackelberg equilibrium. If the firm defaults on its loan, the creditor is exposed to bankruptcy costs. The creditor asks for a positive expected return from lending, which we interpret as the underwriter fee. The bankruptcy costs and the underwriter fee are the capital market imperfections captured in our model and create deadweight costs of external financing for the firm.

In a parsimonious model, we solve for the optimal technology, capacity, production, external borrowing and financial risk management decisions of the firm and the creditor's optimal contracting decision in equilibrium. We characterize a perfect capital benchmark that arises naturally from our framework. In perfect capital markets, the operational and financial decisions decouple and do not interact. Using this perfect market benchmark, we delineate the effect of capital market imperfection costs and analyze the interactions between operational and financial decisions. Our main results are the following:

1. An increase in capital market imperfection costs decreases the the optimal capacity investment level and operational performance of the firm.
   Driver: Higher imperfection costs lead to higher financing costs in equilibrium.

2. In a single product setting, an increase in demand variability decreases the optimal capacity investment level and the operational performance of the firm; this effect does not exist in perfect capital markets.
   Driver: Higher variability leads to higher default risk and hence higher financing costs in equilibrium.

3. In a two-product setting with dedicated technology, an increase in demand variability or correlation decreases the optimal capacity investment level and the operational performance of the firm; optimal capacity investment level and the operational performance of the firm are independent of these parameters in perfect capital markets.
   Driver: An increase in correlation reduces diversification benefits, which leads to a higher default risk and hence higher financing costs in equilibrium.

4. Flexible technology may have negative strategic production switching value; this value does not exist in perfect capital markets.
   Driver: The adjustment in the optimal capacity investment level to exploit production switching leads to a higher default risk and hence higher financing costs in equilibrium.

5. Financial risk management has positive static value and negative strategic value; financial risk management has no value in perfect capital markets.
   Driver: Financial risk management decreases the expected borrowing level of the firm for a given financing cost (static value). It also leads to a loss of revenue for the lender due to the lower expected borrowing level and hence an increase in financing costs in equilibrium (strategic value).

With these results, we contribute to the growing operations management literature that incorporates financial considerations in operational decision making. Our analysis demonstrate undocumented tradeoffs in the capacity and technology investment decisions and provide guidelines for managers concerning capacity management.

This paper brings constructs and assumptions motivated by the finance literature into a classical operations management problem and highlights trade-offs undocumented in this literature. In turn, by modelling operations in more detail than the finance literature, we provide novel insights on issues discussed in this literature. For example, in contrast to arguments summarized in McKay (2003), we show that operational flexibility may increase the financing costs in equilibrium. We add to the argument that financial risk management has positive strategic value (Smith and Stulz 1985) by showing that it can have negative strategic value.

Chapter 4

Operational Hedging: A Review with Discussion

4.1 Introduction

Corporations are faced with a wide variety of risks such as supply-demand coordination risks, exchange rate risks, political risks and disruption risks. Corporate risk management programs aim to systematically manage such risk exposures so as to increase firm value. In the aftermath of serious financial losses by prominent firms and local governments due to inappropriate risk management programs based on financial derivatives, a survey in The Economist (1996, p.18) focuses on "other ways of spreading risk in non-financial companies." In particular, the article discusses "natural hedges" such as financing an operation in local currency, and "operational hedging" such as relocating production facilities to get a better match of costs to revenues. As noted in a recent series of articles in the Financial Times on corporate risk management, "In the past few years, car makers have also been addressing manufacturing risks by reorganizing large chunks of their business to offload risk to suppliers" (Financial Times 2003, p.4). Another example is Microsoft's reliance on temporary workers: "We [Microsoft] count on them [temporary workers] to do a lot of important work for us. We use them to provide us with flexibility to deal with uncertainty" (Los Angeles Times 1997, p.D1 as quoted by Meulbroek 2002b). Such operational flexibility is important for the firm to respond to unexpected shocks in demand, technology or regulation (Meulbroek 2002b). Motivated by the increasing prevalence of operational hedging in corporate-level risk management programs, we provide an extensive overview and synthesis of the existing literature on operational hedging. We start by discussing the rationale behind corporate risk management and tools available for this purpose.

The main objective behind corporate risk management programs is to increase shareholder wealth by enhancing firm value through the management of risk exposures. Paradoxically, building on the seminal work of Modigliani and Miller (1958), classical finance theory asserts that under perfect and complete markets, corporate risk management programs do not add any value: Under these assumptions, the benefits of any risk management activity by firms can be reproduced by shareholders through asset diversification. In other words, risk management cannot create value by undertaking activities that investors can do equally well.

However, there are several rationales motivating corporate-level risk management programs. Market imperfections exist that make volatility costly to firms and that are effectively managed only through firms themselves (Fite and Pfleiderer 1995). The corporate finance literature identifies different market imperfections as reasons for the existence of firm-level risk management: financial distress and bankruptcy costs (Smith and Stulz 1985), corporate taxes (Smith and Stulz 1985), more costly external financing (Froot et al. 1993), and agency problems such as managerial risk aversion (Smith and Stulz 1985) and information asymmetry between managers and shareholders (DeMarzo and Duffie 1995). Aside from these market imperfections, another reason for corporate-level risk management programs is that shareholders hardly hold well-diversified portfolios (for as in the case of family-owned firms). Even if they are well-diversified, shareholders might still prefer corporations to manage their risk exposures in order not to reestablish their portfolios very frequently (Fite and Pfleiderer 1995).

The first step in any risk management activity is the identification and assessment of risk exposure (Bodie and Merton 1998). Firms are exposed to a portfolio of risks, some of which are firm-specific whereas the rest are inherent to capital markets and common to all firms in the economy (market risks). Some of these risks are contingent on asset prices such as interest rates, exchange rates and commodity prices. However, there are other types of risks that mainly stem from firm operations. Kleindorfer and Van Wassenhove (2003) consider risk management in the global supply chain and discuss two broad categories of risk: disruption risk due to accidental or purposeful triggers (e.g. earthquakes, terrorism) and supply-demand coordination risk (e.g. order cancellation, supplier default). According to Billington et al. (2003), uncertainties about demand for products and supply of key inputs are the greatest risks of most manufacturers. These risks create a supply-demand mismatch that results in financial losses.

After determining their risk portfolio, firms have a significant number of tools to put to use in managing their exposures. Taking short or long positions in financial derivatives (forwards, futures, options, swaps etc.), carrying large cash balances, adopting conservative financial policies (Tufano 1996) or holding foreign denominated debt (Geczy et al. 1997) are financial means for risk management. In particular, financial derivatives, tailored contracts written over asset prices such as interest rates, exchange rates and commodity prices, which provide risk transfer between the transacting parties, have been utilized extensively at the firm level through well-developed financial markets for a long time.

Although such financial tools are appropriate for firms that have risk exposures contingent on asset prices, other types of risks stemming from firm operations cannot be managed through the use of financial contracts (Guay and Kothari 2003). In addition to contractual agreements between parties (Cachon 2002), firms engage in operational activities to manage such risk exposures. Investments having real option features are the prevalent instruments used for this purpose. Real options are "opportunities to delay and adjust investments and operating decisions over time in response to resolution of uncertainty" (Triantis 2000). The value of real options is driven not only by timing (through the postponement of operating decisions) but also by scope (by providing a set of alternatives instead of a single choice) (Billington et al. 2003).

Real options are referred to as operational hedging mechanisms in the operations management literature. Operational hedging has been studied in a variety of fields - operations management, finance, strategy and international business. In all fields, operational hedging is discussed in conjunction with financial hedging, and mostly analyzed in a multinational context. The existence of risks that can only be managed operationally (Triantis 2000) means that operational hedging constitutes an important part of firm-level risk management programs: Empirical investigations (Allayannis et al. 2001, Pantzalis et al. 2001) clearly demonstrate that firms do use operational hedges in managing their risks.

Let us demonstrate the role of operational hedging by an example in a multinational framework. A manufacturing firm with production and sales operations in foreign countries is exposed to demand and exchange rate risks. The firm can use financial tools (e.g. forwards) to manage its exposure to exchange rate risks, but these tools are not effective in altering the demand risk exposure. However, postponing the production decision until after more accurate information about demand is acquired buffers against demand uncertainty by better matching supply and demand. This operational decision (postponement), used as a risk hedging device, is an operational hedge of the multinational firm.

Although there are similarities in forms of operational hedging across different academic fields, as we discuss below, we observe that there is no consistent framework on operational hedging that spans these fields. In this paper, we review and provide a synthesis of existing literature on operational hedging from the operations management, finance, strategy and international business fields, and discuss and critique the operational hedging framework developed in operations management in the light of the broader literature.

Two related definitions of operational hedging have been proposed in the operations management literature. We state and discuss these definitions in Section 2.1 where we explore how operational hedging is addressed in the operations management literature. Sections 2.2 and 2.3 do the same for the finance, and strategy and international business literatures, respectively. Thus, Section 2 provides an extensive overview of the forms of operational hedging that appear in the operations management, finance, strategy and international business literatures, which has not appeared in the literature to date.

Section 3 identifies some limitations and inconsistencies of the definitions of operational hedging in operations management, in the light of the broader literature on the topic. In particular, we demonstrate that real options are not the only means of operational hedging, but that there are additional operational tools that firms can employ to mitigate their risks (Section 3.1). In addition, based on the hedging rationale put forward in the finance literature, we argue that real options should not always be considered as operational hedges (Section 3.2). Finally, we show that real options do not necessarily satisfy the type of risk reductions that form the basis of the existing definitions (Section 3.3). Section 4 concludes the paper.

4.2 Literature Review

In this section, we review the literature on operational hedging in operations management, finance, strategy and international business. We only concentrate on operational hedging and therefore do not cover other literature on risk management. In addition, we do not consider contractual agreements for transferring risks (Spinler et al. 2002) as operational hedges, but focus only on operational means of hedging. Finally, we do not review recent research in operations management that incorporates risk aversion or real option valuation methods and refer interested readers to Van Mieghem (2003) and Smith and McCardle (1998) and the references therein, respectively, for reviews of these literatures.

4.2.1 Operations Management

In operations management, there are two streams of research originating from two separate, but conceptually similar, definitions of operational hedging. The first definition, as introduced by Huchzermeier (1991) and quoted in Ding et al. (2005), states that "Operational hedging strategies can be viewed as real (compound) options that are exercised in response to demand, price and exchange rate contingencies faced by firms in a global supply chain context." These options are supply chain network options that are derived from the global coordination of sourcing and/or production decisions. Postponing the logistics decision (Ding et al. 2005), switching production and sourcing strategies contingent on demand and exchange rate uncertainties (Cohen and Huchzermeier 1999), switching among supply chain network structures (Huchzermeier and Cohen 1996), holding excess capacity (Cohen and Huchzemeier 1999) and delaying the final commitment of capacity investments are means of operational hedging. These real options, used as operational hedges, are argued to mitigate the risk exposure in the long run by reducing the downside risk (Cohen and Huchzermeier 1999).

All of the above real options are forms of operational flexibility, which is created through the deployment of excess capacity and/or stochastic recourse. As defined in Cohen and Huchzermeier (1999), operational flexibility is a firm's ability to anticipate and respond to changes in market conditions flexibly by means of the firm's operations. By exercising these options, multinational exploit the volatility in the environment. To explain what this means, consider the example given in Cohen and Huchzermeier (1999): A multinational firm determines the location of production facilities (network structure) but postpones the production quantity decision (logistics decision) until after seeing the demand and exchange rate realizations. Without the postponement option, the firm would choose a given network structure and production quantities and obtain a level of profits. When it has the option to postpone the logistics decision, on the other hand, the firm may choose a different network structure with more facilities (excess capacity). The authors show that the value of the firm may then increase. In other words, real options have value-enhancing capabilities under uncertainty. Note that the postponement option would not have created any value if demand and exchange rate were deterministic. For this reason, the value-enhancing feature of real options under uncertainty is called "exploiting uncertainty." This value increase is achieved without necessarily reducing the volatility of the firm's cash flows. In fact, even in a risk-neutral setting, where volatility of cash flows is not of concern, it may be beneficial to use real options due to their value-enhancing capabilities (Ding et al. 2005).

Huchzermeier and Cohen (1996) analyze operational flexibility, which they define as the ability to switch among different global manufacturing strategy options. Global manufacturing strategy options are created by combining product options (that introduce international supply flexibility) and supply chain network options (that introduce manufacturing flexibility through production capacity and supply chain linkage choices). The authors argue that with operational flexibility, the volatility of firms' cash-flows is not eliminated but exploited, and that this form of operational hedging utilizes the global supply chain network design to mitigate against exchange rate exposure, increasing the value of the firm and decreasing its downside risk.

Cohen and Huchzemeier (1999) illustrate how the deployment of excess capacity can be a source of operational flexibility in global supply chains. They argue that investing in capacity in excess of the aggregate demand forecast provides flexibility in coping with demand uncertainties. Additionally, they focus on the option to postpone the commitment of resources (stochastic recourse) together with the option to switch among different production locations. Through stochastic recourse, the firm discovers the minimum-cost production location depending on exchange rate realizations. Additionally, excess capacity enables the firm to produce more in that location, providing a value-enhancing opportunity in addition to reducing its downside risks.

Postponing the logistics decision is examined by Ding et al. (2005) in a two-stage, single-period model. A multinational firm producing domestically and selling only in a foreign market is exposed to demand and exchange rate risks. In the first stage, the firm commits to the production/capacity level taking into account demand and exchange rate uncertainties. In the second stage, after all the uncertainty is resolved, the firm decides how much to allocate from its domestic capacity to the foreign market. The postponement of the allocation decision until after seeing demand and exchange rate realizations is a real option and constitutes the firm's operational hedging strategy. The authors demonstrate that the allocation option increases the expected utility of both risk-averse and risk-neutral decision makers.

The second definition of operational hedging is found in Van Mieghem (2003). Without referring to real options, but making an analogy with its financial counterpart, financial hedging, Van Mieghem defines operational hedging as "mitigating risk by counterbalancing actions in a processing network that do not involve financial instruments." He lists dual-sourcing, component commonality, having the option to run overtime, dynamic substitution, routing, transshipping, or shifting processing among different types of capital, locations or subcontractors, holding safety stocks and purchasing warranty guarantees operational hedging strategies.

We make several observations concerning this definition. One of the main contributions of this definition is the observation that operational hedging can be employed in the absence of tradable risks, particularly exchange rate risk - as we discuss later, all the other academic fields mostly consider operational hedging in an exchange rate framework. Again departing from the literature, Van Mieghem dos not consider any particular risk measure to formalize the effect of operational hedging in terms of risk mitigation. In addition, the term "counterbalancing actions" is not formalized: criteria to determine whether given actions are counterbalancing are not developed. In our understanding, this term corresponds to investing in more than one resource, or "betting on two horses" (conversation with the author), that is, investing in operational flexibility, similar to the former definition of operational hedging. Observe that, although not explicitly articulated, all the proposed strategies can be viewed as real options. The real option values of these strategies are driven through either timing (postponement of operational decisions) or scope (through providing a set of alternatives instead of a single choice), if not both. Finally, as with real options, counterbalancing actions described by Van Mieghem have a value-enhancing capability and increase expected profit in a risk-neutral setting. This is demonstrated on a two-product, two-stage production system where capacity imbalance is the operational hedging strategy (Harrison and Van Mieghem 1999, Van Mieghem 2003). These papers argue that by purposely unbalancing the capacity vector, i.e. having safety capacity (in excess of the capacity that would be optimal in the deterministic case), firms can hedge against demand uncertainty and increase expected profit. Counterbalancing actions, taken in such a way as to maximize expected profit for a risk-neutral decision maker, are called operational hedges.

4.2.2 Finance

The finance literature has used the term "operational hedging" in the last decade with increasing frequency. It is always discussed in conjunction with its financial counterpart, financial hedging. In the finance literature, operational hedging is the course of action that hedges the firm's risk exposure by means of non-financial instruments, particularly through operational activities.

Similar to the operations management literature, operational flexibility is the major operational hedging strategy discussed in the finance literature. Finance research underlines the value-enhancing capability of this kind of flexibility by referring to its real option features. Even in a risk-neutral setting, creating real option features in an existing investment increases value by providing flexibility in the decision-making process. Since most of the papers are in the context of multinational corporations, operational flexibility in the form of switching production or sourcing locations is the most prevalent type of operational hedging strategy.

In addition to operational flexibility, geographical diversification is discussed as another operational hedging strategy in a multinational context. Geographical diversification is aligning the costs and revenues of a firm so that they are exposed to the same risks. Domestic firms selling to foreign markets can ensure that their production costs and sales revenues are exposed to the same exchange rate uncertainties by opening a production facility in these markets. As in the case of operational flexibility, firms reduce their downside exposures to exchange rate risks by eliminating the negative effect of appreciated local currency (in the form of higher production costs). However, different from operational flexibility, firms also sacrifice the gains in the upside by forgoing the positive effect of depreciated currency (in the form of lower production costs). Therefore, geographical diversification reduces the total variability of cash flows. Chowdry and Howe (1999) consider opening a production facility in a foreign market as the operational hedging strategy of multinational firms without differentiating between geographical diversification and operational flexibility. They analyze the conditions under which firms engage in financial and operational hedging strategies with respect to exchange rate and demand risks. They state that by having plants in several countries, multinationals can align their costs and revenues besides shifting production among these locations. They argue that the facility location decision is considered to be an operational hedging strategy only when firms are concerned with the variability of their operating profits.

Hommel (2003) considers geographical diversification and operational flexibility in the form of a real switching option as two separate operational hedging strategies. He investigates the incentives of firms to hedge currency risk with financial and operational (there, "operative") means in a multinational context. The hedging motivation is introduced through a minimum profit constraint such that firms have incentives to hedge their payoffs to satisfy this constraint. He argues that operational flexibility is employed as a hedging device when the exchange rate and demand volatility are sufficiently large (in that case the minimum profit constraint is violated); otherwise it serves as a value driver to enhance expected profits.

These papers emphasize that because operational flexibility can be used for a purely value-enhancement motive, it is considered to be an operational hedging strategy only when there is a risk hedging motive for employing it. Generally speaking, operational actions are considered to be operational hedges if they are taken in order to reduce a risk measure of concern. In particular, if firms care about downside risk (e.g. having a minimum profit constraint), then operational hedges mitigate risk through a reduction in the downside exposure. If variance of the payoffs is the risk measure under consideration (e.g. having a convex tax schedule), then operational hedges mitigate risk through a reduction in variance.

In empirical research in risk management, operational hedging strategies are always studied in conjunction with financial derivatives in an exchange rate or commodity setting. Geographical diversification and operational flexibility are the operational hedging strategies implemented through different operational decisions. This field mainly investigates the substitutability or complementarity of operational and financial hedging instruments and tests whether firms use risk management activities under different risk management motives.

Fok et al. (1997) consider locating production facilities in major foreign markets to minimize foreign exchange rate exposure, and choosing a technology to minimize exposure to commodity price risk to be production-originated hedging instruments of multinational firms. Although the term "operational hedging" is not used, the former is simply geographical diversification whereas the latter is similar to a product differentiation strategy (Miller 1998), which is a type of operational flexibility. In a multinational context, Allayannis et al. (2001) proxy the operational hedging of multinationals by the level of geographic dispersion (the location of subsidiaries across multiple countries or regions) without differentiating between geographical diversification and operational flexibility. They investigate both financial and operational exchange rate risk management strategies of firms, and demonstrate how much each strategy contributes to the overall goal of mitigating risk and improving shareholder value.

In a similar framework, Doukas and Padmanabhan (2002) consider the intangible assets of firms to be operational hedging devices with respect to political risks. The authors argue that by having high levels of intangible assets, firms can compensate the loss due to the political interruption of a host government using their other assets (for example, in other countries). Observe that high levels of intangible assets provide flexibility in terms of shifting resources among countries or businesses; this is another form of operational flexibility.

In a commodity setting, Petersen and Thiagarajan (2000) focus on gold mining firms. These firms, by adjusting their mining strategies as a function of gold price, create cost structures that positively correlate with the price of gold. Operational flexibility, created by the ability to adjust cost structures, is their operational hedging strategy, and creates a natural hedge against gold price exposure.

In summary, the finance literature defines operational hedging as mitigating firms' risks by operational means. Operational flexibility achieved through various operational means (ability to shift production, transferring technologies, product differentiation etc.) and geographical diversification are the operational hedges of firms utilized in conjunction with financial hedges. Compared to their financial counterparts, operational hedges require higher levels of capital investment (opening a production facility), but create longer term hedges against risk exposures including risks that are not contingent on asset prices (e.g. demand risks, political risks).

4.2.3 Strategy and International Business

Research in the strategy field provides a more comprehensive and complete discussion of diversification and operational flexibility from different perspectives. Diversification is defined as having different lines of business through mergers and joint ventures (Wang and Lim 2003), of which geographical diversification is one type.

Kogut (1985) analyzes diversification and operational flexibility as risk management tools of multinationals. He examines how operational flexibility and diversification change the risk profiles of firms. He argues that an operational decision (the sourcing policy in this case) can create three different types of risk profile: speculative, hedged and flexible. The speculative profile is betting on one site mainly to benefit from economies of scale in operations. By matching the exchange rate exposure on the cost side with that on the profit side, the firm can create a hedged risk profile. This approach corresponds to the geographical diversification strategy discussed in the finance literature. Finally, a flexible risk profile created through operational flexibility permits the firm to exploit uncertainties by creating real options. Operational flexibility creates both arbitrage (exploitation of differences between markets such as production switching) and leverage (enhancing strategic position such as increased bargaining power in negotiations with local governments) opportunities for multinationals.

Miller (1998) says that strategic hedges, which he defines as real options, can be used to hedge corporate downside risk. He discusses operational flexibility and diversification as strategic hedges: Similar to operational flexibility, diversification is claimed to have real option benefits. In particular, diversification into new product or geographic markets has an option value through creating growth options (Kogut 1991, Kogut and Kulatilaka 1994). Other than aligning costs and revenues, by opening a production facility in a foreign country, firms can exploit being in that market by the cost effectiveness of launching new products in the same market. Under operational flexibility, Miller lists developing in-house capacity to produce inputs when a firm has negative exposure to input prices, vertical integration of a key supplier when the firm faces the price risk of a non-commodity input, reducing the price elasticity of demand through product differentiation, and increasing customer brand loyalty and switching costs when the firm faces price competition.

In the international business literature, Pantzalis et al. (2001) define operational hedging as the firm's operational decisions (related to marketing, production, sourcing, plant location, treasury) that are best suited to managing the exchange rate exposure on the firm's competitive position across markets. Without using the term "geographical diversification," they consider the shifting of production to offset price changes with local cost changes to be an operational hedging strategy. As another operational hedging strategy, they describe the operational flexibility of multinationals in the form of shifting production and transferring resources within their network. Carter et al. (2003) define operational hedging strategies as a combination of production and marketing strategies across the firm's operating units developed to manage long-term exposures. Other than geographical diversification, they discuss real option type operational hedging strategies such as shifting sourcing or production, exploiting growth-options, having pricing flexibility and abandoning foreign markets. Observe that all of these strategies are again types of operational flexibility.

In summary, the strategy literature focuses on operational flexibility and diversification as risk management tools without defining them as operational hedges. Operational flexibility achieved through several operational means (developing in-house capacity, product differentiation, keeping excess capacity etc.) creates both arbitrage and leverage opportunities for multinational firms. In addition to aligning costs and revenues, real option benefits of geographical diversification in the form of growth options are discussed. The international business research, similar to the finance literature, focuses on operational flexibility and geographical diversification as long-term operational hedges of multinationals against exchange rate exposures.

4.2.4 Summary

The operations management literature views operational hedging strategies as real options, originating from two separate, but not conceptually different definitions. According to this view, operational hedging is investing in operational flexibility, which acts as a value driver for the firm even in the risk neutral setting. The risk mitigation connotation that the word "hedging" brings is addressed by claiming that downside risk is reduced in the first definition, whereas such a justification is not put forward in the second. Other fields define operational hedging as operational means of reducing firms' risk exposures. Operational flexibility created through real options and geographical diversification are the main operational hedging strategies studied in these literatures. Compared with financial hedging, operational hedging requires higher levels of capital investment (opening a production facility), but creates long-term hedges against risk exposures including risks that are not contingent on asset prices (such as demand risks, political risks). In particular, operational flexibility has a value creation capability through arbitrage and leverage opportunities. Therefore, in finance, this kind of flexibility is considered to be an operational hedging strategy only when there is a risk hedging rationale for using it.

4.3 Discussion

In this section, we evaluate and critique the existing definitions of operational hedging in operations management in the context of the broader literature on the topic. Recall that in operations management, operational hedging strategies are defined as (i) real options mitigating downside risk or (ii) counterbalancing actions that do not involve financial instruments, which we interpreted as also being real options. The next sub-section discusses a limitation of these definitions.

4.3.1 Operational hedging strategies are not only real options.

In this section, we illustrate some operational decisions that mitigate firms' risk exposures, and should therefore be considered operational hedging strategies. However, these decisions do not have real option characteristics, and cannot be captured by existing definitions.

A basic example of non-real-option type operational hedging strategies is geographical diversification as discussed in the finance literature: Domestic firms selling to foreign markets can ensure that their production costs and sales revenues are realized in the same currency and are thus exposed to the same exchange rate uncertainty by opening a production facility in these markets. As discussed in Section 2.2, this strategy reduces the negative effect of appreciated local currency but forgoes the positive effect of depreciated local currency. Since the exchange rate exposure is mitigated by operational means, geographical diversification in the sense of aligning costs and revenues is an operational hedging strategy, but it is not a real option: It does not provide operational flexibility.

Besides geographical diversification, there are other operational strategies that provide risk reduction or risk-sharing benefits, and that do not have real option characteristics: (i) Instead of transferring the exposure to the counterparty, firms can take actions to reduce the overall risks taken by both parties; (ii) Some operational decisions might result in implicit risk-sharing between parties without relying on contractual agreements.

For example, as stated in Meulbroek (2002b), one of the major risks for Disney Corporation is the weather risk, since bad weather significantly reduces the number of visitors to Disney theme parks. However, by locating the theme park in a warm and sunny region (such as Florida), Disney created a natural hedge against weather risks. The location decision reduced the overall exposure of both the firm and its customers (both parties) to the weather risk by reducing the likelihood of unfavorable states of nature (bad weather). Another way of reducing Disney's weather risk through operational means is locating smaller-size theme parks close to major population centers (Meulbroek 2002a). This type of theme park draws single-day visitors rather than multiple-day visitors, inducing a change in perception of weather risk among customers: For short-term visits, customers care less about bad weather risk, and are willing to bear the risk. The location decision provides implicit risk-sharing benefits since Disney shares the weather risk with its customers who internalize and bear it. In contrast to the first type of location decision, Disney creates an operational hedge by reducing the consequences of unfavorable states of nature, and not by altering the likelihood of these states.

These strategies, while they mitigate firms' risk exposures, neither have real option characteristics nor are counterbalancing actions, and are therefore not covered by the current definitions of operational hedging in the operations management literature.

4.3.2 Real options are operational risk management tools, but not necessarily hedging tools.

In the previous section, we argued that real options are not the only operational means in hedging firms' risks. In this section, we argue that real options should not be equated with operational hedging: Although real options are operational risk management tools, they are not necessarily used as risk hedging devices. The operations management literature sees real options as analogs of financial options, which are risk hedging devices, and for this reason considers them to be operational risk hedging devices. This creates an inconsistency between the way in which real options are discussed in the operations management and the other literatures.

Risk management in the broad sense is not equivalent to risk hedging (Triantis 2000, MacMinn 2002). Instead, it is the creation or preservation of firm value through managing exposures. One example for risk management without hedging is speculation with financial derivatives. In the finance literature, financial markets are assumed to be efficient and therefore there is typically no room for arbitrage. Nevertheless, when there are arbitrage opportunities, firms can choose to speculate on financial markets to create value (Moschini and Lapan 1995). In this case, firms can exploit their risk profiles and take positions that increase their exposures.

Similarly, in exercising a real investment opportunity, it can be in their best interest for firms to increase their risk exposures. In particular, real options have value-enhancement capabilities in addition to their hedging benefits: Creating real option features in an investment provides flexibility in the decision-making process. For this reason, real options are operational means of managing risks, but they are not necessarily used as operational hedging strategies that decrease the risk exposure. Indeed, the finance literature considers real options to be operational hedging mechanisms only when firms utilize them as a result of concerns about the volatility of their payoffs in the presence of market imperfections (Chowdry and Howe 1999, Hommel 2003).

In the operations management literature, real options are called operational hedging devices even in a risk-neutral setting (a setting typically used in this literature) because they increase expected profit by exploiting uncertainty. As we said above, the finance literature requires the firm to have a risk minimization motive to consider an operational action to be an operational hedge. Therefore, in the finance literature, counterbalancing capacities in a risk-neutral world and in the absence of market imperfections (as in Van Mieghem 2003) would not be considered as hedging devices.

4.3.3 Real options do not necessarily decrease the downside risk or variance of total payoff.

Operational hedges are said to reduce the downside risk of the firms (Huchzermeier and Cohen 1996). However, following the previous section's discussion, we demonstrate that real options do not necessarily decrease the downside risk (or the variance) of firms' payoffs.

The argument that real options enable firms to limit their downside risks while keeping the upside potential alive (Triantis 2000) is valid when all else is kept equal, that is, the only change in the environment is the existence of real options. This is very intuitive: Firms exercise their real options under unfavorable states of nature, and truncate their downside losses by utilizing these opportunities. However, the existence of real options might alter other operational decisions of firms. In that case, after exercising the real option and optimally resetting the levels of decision variables, the downside risk exposure or variance of this new payoff might be higher than that without the real option. Put differently, as argued in the previous section, after exercising their real options, firms may optimally adjust their operational decisions to exploit more of the underlying uncertainties.

To illustrate this, we consider the multinational firm that makes capacity and logistics decisions with or without the allocation option (Ding et al. 2005). The authors call this real option an operational hedge, referring to the first definition of operational hedging by Huchzermeier and Cohen (1996). In their model, the multinational firm producing domestically and selling only in a foreign market has to decide the production quantity and how many of those units to transfer to the market (the logistics decision). The allocation option refers to the option of delaying the logistics decision until after the demand and exchange rate uncertainties have been resolved; otherwise the quantity shipped equals the quantity produced. Assume that without the allocation option, the expected unit revenue is less than the production and logistics cost per unit. Then the firm optimally chooses not to produce at all. If it has the option to postpone the logistics decision, the firm calculates the expected value of the minimum of incremental profit (unit price minus unit transportation cost) and zero, since the firm has the option not to transfer any quantity if the incremental profit is negative. If the expectation is larger than the unit production cost, then the firm optimally commits to a positive production quantity. Notice that without the allocation option, the operating cash flows are constant (zero), but the existence of postponement creates a random cash flow stream that may involve negative realizations. Employing expected loss (Huchzermeier and Cohen 1996, Szego 2002) as the downside risk measure, which is the expected value of negative deviations from a reference level, and setting the reference level to zero, we conclude that the existence of the allocation option increases the downside risk of the firm. Other examples demonstrating the same phenomenon can easily be developed.

In the operations management literature, operational hedging strategies are said to decrease the downside risk, and postponing the logistics decision is one of the cited operational hedging strategies (Cohen and Huchzermeier 1999). However, as we illustrated above, the downside risk of the firm does not necessarily decrease when operational hedging strategies impact other operational decisions. And the existence of additional operational decisions other than exercising real options is common in the operations management literature. We conclude that care must be taken when claiming that strategies that are classified as operational hedging reduce the downside risk: they are guaranteed to decrease the downside risk only if no other operational decisions are modified due to the existence of the real option.

Although one school of thought in the finance literature argues that the primary goal of corporate risk management programs is to eliminate the probability of costly lower-tail outcomes, i.e. the downside risk (Stulz 1996), variance is also utilized as a risk measure (Chowdry and Howe 1999). The operations management literature has recently incorporated risk aversion through mean-variance type utility functions (Chen and Federgruen 2000, Gaur and Seshadri 2005) and operational hedging has been analyzed in the mean-variance framework (Ding et al. 2005, Van Mieghem 2003). Since hedging is mitigating the risk exposure, one may expect an operational hedge to decrease this risk measure. However, as mentioned in Ding et al. (2005), when the exchange rate and demand distributions are correlated, then the allocation option may in fact increase the variance of the firm's operating profits. In this case, not only the existence of additional operational decisions, but the use of variance as the risk measure drives this result: a measure of dispersion (variance, in this case) can be adopted as a risk measure only if the distribution is symmetric (Szego 2002). Moreover, variance is the perfect indicator of risk when comparing two normal or uniform distributions (Eeckhoudt and Gollier 1995, p.82); and is not applicable to newsvendor-based models such as in Ding et al. (2005) and Van Mieghem (2003).

4.4 Conclusion

Intense market competition and high levels of economic and technological uncertainties inherent in the business environment fuel the growth in corporate-level risk management programs. According to the finance literature, there are several sources of market imperfections that make volatility costly to firms and that can be managed through firm-level risk management activities. Financial instruments are effective in managing the exposures dependent on asset prices such as exchange rate, interest rate and commodity price. However, many firms have risks stemming from their operations that are not tradable in capital markets by means of financial contracts. For this reason, operational hedging - drawing on operational tools to hedge risks - constitutes an important component of firm-level risk management programs. Indeed, empirical research shows that firms employ operational means to manage their risk exposures (Allayannis et al. 2001, Pantzalis et al. 2001).

Operational hedging has been discussed in a variety of fields. Operations management research employs two separate, but conceptually similar, definitions of operational hedging. However, these definitions do not capture the complete range of operational hedging strategies discussed in the broader literature. According to one definition, operational hedges are referred to as real compound options of multinational firms that decrease the downside risk. The second definition states that operational hedging consists of non-financial counterbalancing actions in the processing network. As discussed in Section 2.1, both definitions refer to real options (that create operational flexibility) as the primary form of operational hedging strategies. However, there exist other operational activities mitigating firms' risks, as discussed in other academic fields, which do not carry real option characteristics. In particular, geographical diversification and operational decisions that provide risk-sharing benefits are non-real options type operational hedging strategies.

Moreover, we establish some inconsistencies in the definition of operational hedging between the operations management and the finance literatures, as well as within the operations management field. Operational flexibility, because of its real option characteristic, has a direct value-enhancing capability. Consequently, the finance literature refers to operational flexibility as a hedging tool only when firms do care about hedging their risks; otherwise it is considered to be a risk management device. However, one definition of operational hedging in the operations management literature considers counterbalancing capacities in a risk-neutral and perfect-market setting as operational hedging, which is not consistent with other fields. The other operational hedging definition considers operational hedging strategies as means of reducing downside risks. However, when there are additional operational decisions to take, the availability of real options might induce firms to increase their downside risk or variance of total payoffs after optimally re-selecting levels of these operational decisions.

In summary, while the existing definitions of operational hedging in operations management capture the fundamental principles of operational hedging, they are not complete or fully consistent with the usage in other academic fields. We believe that there is room in operations management for an operational hedging framework that incorporates and unifies findings from other fields.

Chapter 5

Conclusions and Future Research Directions

We have considered three essays that shed light on the interplay between a number of operational and financial decisions of the firms in capacity investment setting. Chapter 2 has investigated the integrated risk management portfolio choice of the firm that consists of operational (flexible technology) and financial (forward contracts) means of risk management. We have characterized the optimal risk management portfolio choice as a function of firm size, technology and financial risk management costs, product market (demand variability and correlation) and capital market (external financing costs) characteristics. We have argued that several of the controversial empirical observations about financial risk management practices of the firms can be explained by looking at the interplay between operational and financial decisions. Our modelling framework helps us to understand how to design the integrated risk management portfolio and the value and limitation of each risk management strategy.

Chapter 4 extends the model considered in Chapter 3 by endogenizing the external financing costs and investigates the technology choice and the capacity investment decision of the firms in imperfect capital markets and analyzes the robustness of the traditional insights that implicitly assume perfect capital markets. We demonstrate that imperfections in the capital markets create a strategic effect by affecting the equilibrium level of financing costs and that this strategic effect may reverse our traditional insights on capacity investment. For example, even with identical cost structures, it may not be in the best interest of the firm to invest in flexible technology over dedicated technology because flexible technology may have a negative strategic risk-pooling value that can dominate the always positive static risk-pooling value.

Chapter 5, building on the insights of the first two chapters, discusses several characteristics of the definitions of "operational hedging" proposed in the literature. This chapter demonstrates different means of operational hedging and several properties of these operational hedges. We highlight that firms using operational hedges may end up having more volatile cash flow streams but still create value by exploiting the product market related risks.

In the rest of this chapter, we discuss some of the implications that can be derived from this thesis and propose some future research directions.

This thesis clearly demonstrates that financial risk management is not a panacea and firms can rely only on operational flexibility to manage their risk exposures. In the aftermath of serious losses of prominent firms due to malfunctioning of financial risk management programs, firms become more sceptic about financial derivatives. We illustrate that financial risk management should be seen as an integral part of the firm's overall investment portfolio. Even if there is no significant cost associated with financial risk management, firms can be better of by not engaging in financial risk management to create more value in the operational market. Firms should also incorporate the effect of engaging in financial risk management on the external financing costs. Engaging in financial risk management may increase the financing costs of the firms as we have demonstrated.

Second, our analysis clearly illustrates that there are significant differences between the risk management portfolios of different firms. The underlying drivers may work in opposite directions for capital intensive and non-capital intensive technology investments. Firms should not stick on a particular portfolio but should evaluate the value of certain risk management portfolio choice when considering different investment profiles.

Finally, an important point to consider is the effect of financial decisions of the technology choice of the firm in imperfect capital markets. Firms should incorporate financial considerations when making technology decisions, especially if the technology investment requires external borrowing and the capital markets have significant transaction costs. Ignoring, the strategic effect of technology decision on the financing costs may lead to a mis-valuation of a particular technology. An important result arises from this thesis is that higher flexibility may not be beneficial for the firms even if it is costless if there are financial frictions in the capital markets. These are very important lessons for firms that are exposed to these frictions.

Next, we propose some future research directions related to the interaction of operational and financial decisions from different perspectives. First, future research should focus on empirical research to further understand this interaction. On the risk management side, this thesis opens new empirical avenues. The existing literature on risk management typically does not capture operational aspects such as characteristics of different technologies and product market characteristics. As demonstrated by our analysis, these can have a significant effect on the risk management portfolio and generally have opposite effects for large and small firms. The distinction we make between large and small firms (or equivalently, between capital intensive and non-capital intensive industries), and our results related to the effect of technology and product market characteristics on the risk management portfolio provide new hypotheses that can be tested empirically. For example, we expect to see that large firms engage in financial risk management less frequently than small firms in highly positively correlated markets. We also expect to see a positive relation between fixed technology costs and the frequency of engaging in financial risk management for large firms and a negative relation for small firms.

In the empirical line of future research, the biggest challenge is to define empirical proxies to capture operational characteristics of firms. For example, traditional empirical literature uses the network structure of the firm as a proxy for operational hedging. The more dispersed firms are accepted to be more operationally hedged. However, this dissertation and the related research in the OM literature highlight that the firm can be more operationally hedged even if the firm is a local firm because of other operational flexibilities. This line of empirical research should start with providing better proxies under the light of the findings of this dissertation.

Second, an interesting avenue for future research would be to consider different financing policies from a supply chain perspective. The main research question is which party in the supply chain is more favorable for financing the operations for the whole chain? Should we centralize the financing decision and exploit economies of scale in financing? Should the downstream do the whole borrowing and act as a trade-creditor for the upstream party? When do we have the opposite picture? In practice, we have three type of empirical observations where both parties borrow on their own from external parties or they centralize the borrowing at one of the agents. What are the main drivers of this pattern?

Finally, as a follow up to the first chapter, the competitive effects of the operational and financial risk management tools can be analyzed. In practice, because of financial regulations, firms should disclose their financial risk management decisions to other parties. In this case, anecdotal evidence shows that this disseminates information about the firm's operations to other parties. One advantage of the operational tools is that they are not common information. Different firms have different operational flexibilities inherent in their system that are not observable by the competitors. The interaction of operational and financial risk management tools gain an interesting dimension when we consider this information advantage. The additional value of operational risk management can be characterized in an asymmetric information modelling framework.

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Technical Appendix I

Appendix A

Proof of Proposition 1: We start by formulating the stage 2 optimization problem. Let $\Gamma_T\left(\mathbf{K_T}, e_T, \tilde{B}, \tilde{\boldsymbol{\xi}}\right)$ denote the optimal stage 2 operating profit as a function of the state vector $(\mathbf{K_T}, e_T, \tilde{B}, \tilde{\boldsymbol{\xi}})$. Since we assume production is costless, this profit is equal to the maximum sales revenue that can be obtained with the existing capacity.

In stage 1, the firm will have observed the budget realization $\tilde{B}$ and borrowed e_T to invest in capacity level $\mathbf{K_T}$. The remaining cash holdings of $\tilde{B} + e_T - c_T\mathbf{1}'\mathbf{K_T} - F_T$, non-negative by construction, will have been invested into a cash account with return $r_f(= 0)$.

Two outcomes are possible in stage 2: If the firm's final cash position (operating profits and cash account holdings) is sufficient to cover the face value of the loan, i.e. $\Gamma_T(\mathbf{K_T}, e_T, \tilde{B}, \tilde{\xi}) + (\tilde{B} + e_T - c_T\mathbf{1}'\mathbf{K_T} - F_T) \geq e_T(1 + a)$, then the firm does not default; otherwise, it does. If the firm does not default, it repays the face value of its loan and liquidates the non-pledged technology and the physical assets, generating $\gamma_T F_T$ and P, respectively. If the firm defaults, the cash on hand and the ownership of the collateralized physical asset are transferred to the bank. The firm receives the salvage value of the technology $\gamma_T F_T$ and the cash $R(\mathbf{K_T}, e_T, \tilde{B}, \tilde{\xi})$ remaining after the face value of the loan is deducted from its seized assets. We write

$$R(\mathbf{K_T}, e_T, \tilde{B}, \tilde{\boldsymbol{\xi}}) = P + \Gamma_T(\mathbf{K_T}, e_T, \tilde{B}, \tilde{\boldsymbol{\xi}}) + (\tilde{B} + e_T - c_T\mathbf{1}'\mathbf{K_T} - F_T) - e_T(1 + a), \quad (5.1)$$

where we invoke the assumptions that any additional fees in the default state (e.g. bankruptcy fee) are borne by the creditor as out-of-pocket expenditures, and that the loan is fully-collateralized by the physical asset.

Since the shareholders are risk neutral and the risk-free rate is 0, the stage 2 equity value can be written as the sum of the individual components cash flows, regardless of when they are realized:

$$\Pi_T\left(\mathbf{K_T}, e_T, \tilde{B}, \tilde{\boldsymbol{\xi}}\right) = \begin{cases} \Gamma_T(\mathbf{K_T}, e_T, \tilde{B}, \tilde{\boldsymbol{\xi}}) + (\tilde{B} + e_T - c_T\mathbf{1}'\mathbf{K_T} - F_T) & \text{if no default} \\ -e_T(1+a) + \gamma_T F_T + P & \\ \gamma_T F_T + R(\mathbf{K_T}, e_T, \tilde{B}, \tilde{\boldsymbol{\xi}}) & \text{if default} \end{cases} \tag{5.2}$$

Inspecting (5.2) reveals that the equity value can simply be written as

$$\Pi_T\left(\mathbf{K_T}, e_T, \tilde{B}, \tilde{\boldsymbol{\xi}}\right) = \Gamma_T(\mathbf{K_T}, e_T, \tilde{B}, \tilde{\xi}) + (\tilde{B} + e_T - c_T\mathbf{1}'\mathbf{K_T} - F_T) + \gamma_T F_T - e_T(1+a) + P \tag{5.3}$$

regardless of whether the firm defaults or not. Obtaining this unique functional form is essential in preserving tractability and in deriving closed-form expressions for the firm's capacity, technology and financial risk management decisions for a subset of parameter levels.

The production decision only affects the operating profit Γ in (5.3), so optimizing the stage 2 equity value is equivalent to the following optimization problem:

$$\max_{\mathbf{Q}\in\Theta_T} \mathbf{Q}'\mathbf{p}(\mathbf{Q};\tilde{\boldsymbol{\xi}}) = \max_{\mathbf{Q}\in\Theta_T} \tilde{\boldsymbol{\xi}}'\mathbf{Q}^{1+\frac{1}{b}}. \tag{5.4}$$

Here, $\mathbf{p}(\mathbf{Q};\tilde{\boldsymbol{\xi}})' = \left(p(q_1;\tilde{\xi}_1), p(q_2;\tilde{\xi}_2)\right)$, $\Theta_F \doteq \{\mathbf{Q} : \mathbf{Q} \geq \mathbf{0}, \mathbf{1}'\mathbf{Q} \leq \mathbf{K_F}\}$ and $\Theta_D \doteq \{\mathbf{Q} : \mathbf{Q} \geq \mathbf{0}, \mathbf{Q} \leq \mathbf{K_D}\}$ are the feasibility sets for production quantity levels for each technology T.

Let $f(\mathbf{Q}) \doteq \tilde{\boldsymbol{\xi}}'\mathbf{Q}^{1+\frac{1}{b}}$ and $\mathbf{Q_T^*}$ denote the optimal production vector that solves (5.4) for technology $T \in \{F, D\}$. It is easy to establish that $f(\mathbf{Q})$ is strictly concave in $\mathbf{Q}' = (q_1, q_2)$. Since the constraints are linear, KKT conditions are necessary and sufficient for optimality and $\mathbf{Q_T^*}$ is unique. Since $\frac{\partial f}{\partial q_i} = (1 + 1/b)\tilde{\xi}_1 q_i^{1/b} > 0$, and with $b \in (\infty, -1)$, $\lim_{q_i \to 0^+} \frac{\partial f}{\partial q_i} \to \infty$, the non-negativity constraints will be non-binding and the capacity constraint will be binding at optimality. With the dedicated technology, this yields $\mathbf{Q_D^*} = \mathbf{K_D}$ and

$$\Gamma_D\left(\mathbf{K_D}, e_D, \tilde{B}, \tilde{\boldsymbol{\xi}}\right) = f(\mathbf{Q_D^*}) = \tilde{\boldsymbol{\xi}}'\mathbf{K_D}^{1+\frac{1}{b}}.$$

With the flexible technology, according to the KKT conditions, $\mathbf{Q_F^*}$ solves $\left.\frac{\partial f}{\partial q_1}\right|_{q_F^1} = \left.\frac{\partial f}{\partial q_2}\right|_{K_F - q_F^1}$. After some algebra, we obtain $\mathbf{Q_F^*} = \frac{K_F}{\tilde{\xi}_1^{-b} + \tilde{\xi}_2^{-b}}\ \tilde{\boldsymbol{\xi}}^{-b}$ and

$$\Gamma_F\left(K_F, e_F, \tilde{B}, \tilde{\xi}\right) = f(\mathbf{Q_F^*}) = \frac{K_F^{1+\frac{1}{b}}}{\left(\tilde{\xi}_1^{-b} + \tilde{\xi}_2^{-b}\right)^{1+\frac{1}{b}}}\left[\tilde{\xi}_1^{-b} + \tilde{\xi}_2^{-b}\right] = \left(\tilde{\xi}_1^{-b} + \tilde{\xi}_2^{-b}\right)^{-\frac{1}{b}} K_F^{1+\frac{1}{b}}.$$

Defining $\mathbf{N_F} \doteq \left(\tilde{\xi}_1^{-b} + \tilde{\xi}_2^{-b}\right)^{-\frac{1}{b}}$ and $\mathbf{N_D} \doteq \tilde{\boldsymbol{\xi}}$ and substituting Γ_T in (5.3) yields the expression for the optimal equity value Π_T:

$$\Pi_T\left(\mathbf{K_T}, e_T, \tilde{B}, \tilde{\boldsymbol{\xi}}\right) = \mathbf{N_T'}\mathbf{K_T}^{1+\frac{1}{b}} + (\tilde{B} + e_T - c_T\mathbf{1'K_T} - F_T) + \gamma_T F_T - e_T(1+a) + P \quad (5.5)$$

■

Proof of Proposition 2: We start by formulating the stage 1 optimization problem. The optimal expected (stage 1) equity value of the firm, $\pi_T(\tilde{B})$, is given as follows:

$$\pi_T(\tilde{B}) = \begin{cases} \max\left\{\Psi_T(\tilde{B}), \tilde{B} - (1-\gamma_T)F_T + P\right\} & \text{if } \tilde{B} + E > F_T \\ \tilde{B} - (1-\gamma_T)F_T + P & \text{if } \tilde{B} + E \leq F_T \end{cases} \quad (5.6)$$

where

$$\begin{aligned} \Psi_T(\tilde{B}) \quad &= \quad \max_{\mathbf{K_T},\, e_T} \tilde{B} + e_T - (c_T\mathbf{1'K_T} + F_T) - (B + e_T - c_T\mathbf{1'K_T} - F_T) + \mathbf{E}\left[\Pi_T\left(\mathbf{K_T}, e_T, \tilde{B}, \tilde{\boldsymbol{\xi}}\right)\right] \\ \text{s.t.} \quad & \quad e_T \geq c_T\mathbf{1'K_T} + F_T - \tilde{B} \\ & \quad e_T \leq E_T \\ & \quad \mathbf{K_T} \geq \mathbf{0}, \quad e_T \geq 0. \end{aligned} \quad (5.7)$$

We start with explaining the formulation of the optimization problem (5.7). The firm has available budget $\tilde{B}$ and borrows e_T from the creditor. Out of this sum $\tilde{B}+e_T$, the firm invests $c_T\mathbf{1'K_T}+F_T$ in capacity and places the remainder $(B+e_T-c_T\mathbf{1'K_T}-F_T)$ into the cash account. The return from the cash account and the operating profits from the capacity investment are included in the expected value of the equity in stage 2, $\mathbf{E}\left[\Pi_T\left(\mathbf{K_T}, e_T, \tilde{B}, \tilde{\boldsymbol{\xi}}\right)\right]$. Using (5.3), the objective function can be rewritten as $\tilde{B}+P-(1-\gamma_T)F_T+\Gamma_T(\mathbf{K_T}, e_T, \tilde{B}, \tilde{\boldsymbol{\xi}})-c_T\mathbf{1'K_T}-ae_T$. Here, the first three terms are equal to the equity value of the firm if the firm does nothing (does not borrow and does not invest). Note that since the firm has already committed to technology T, the fixed cost F_T is incurred even if $\mathbf{K_T} = \mathbf{0}$. The last three terms are the net profit derived from borrowing and investing in capacity.

The first constraint ensures that the amount of external borrowing is greater than the difference between the cost of the investment and the available budget, otherwise the investment is not feasible. The second constraint states that the external borrowing is less than the credit limit (E) of the firm.

Equation (5.6) states the firm will either choose a positive capacity level in stage 1 or do nothing (not borrow and not invest in capacity). The former will be the case when the optimal capacity investment level obtained in (5.7) is positive, and this solution dominates doing nothing; with $\pi_T(\tilde{B}) = \Psi_T(\tilde{B})$. In the latter case, the equity value of the firm is $\tilde{B} + P - (1-\gamma_T)F_T$, with $\mathbf{K}_{\mathbf{T}}^*(\tilde{B}) = \mathbf{0}$ and $e_T^*(\tilde{B}) = 0$. This is the optimal solution if (i) the budget plus the credit limit is insufficient (or only sufficient) to cover the fixed cost of investment ($\tilde{B} + E \leq F_T$), so the firm liquidates the physical asset and salvages the technology; or if (ii) the budget plus credit limit is sufficient to cover the fixed cost, but the firm optimally chooses not to invest in capacity ($\tilde{B} + P - (1-\gamma_T)F_T > \Psi_T(\tilde{B})$ when $\tilde{B}+E \geq F_T$). Note that if $\mathbf{K_T} = \mathbf{0}$ in the optimal solution of (5.7), the formulation in (5.7) forces the firm to (suboptimally) borrow $E - \tilde{B}$, but the optimal objective function value is then dominated by $\tilde{B} + P - (1-\gamma_T)F_T$, the value of doing nothing, so the joint formulation in (5) and (6) yields the correct optimal solution.

Since $a > 0$, the firm optimally does not borrow if it does not invest in capacity ($e_T = 0$ if $\mathbf{K_T} = \mathbf{0}$) and only borrows exactly enough to cover the capacity investment when this investment level is positive ($e_T = \left(c_T \mathbf{1}'\mathbf{K_T} + F_T - \tilde{B}\right)^+$ if $\mathbf{K_T} > \mathbf{0}$). Substituting Π_T from (5.3) and Γ_T from Proposition 1 in (5.7), we obtain the equivalent formulation

$$\begin{aligned}
\Psi_T(\tilde{B}) = \max_{\mathbf{K_T}} \quad & \tilde{B} - c_T\mathbf{1}'\mathbf{K_T} - (1-\gamma_T)F_T - a\left(c_T\mathbf{1}'\mathbf{K_T} + F_T - \tilde{B}\right)^+ + \mathbf{E}\left[\mathbf{N_T}\right]'\mathbf{K_T}^{1+\frac{1}{b}} + P \\
\text{s.t.} \quad & c_T\mathbf{1}'\mathbf{K_T} + F_T - \tilde{B} \leq E \qquad\qquad (5.8)\\
& \mathbf{K_T} \geq \mathbf{0}.
\end{aligned}$$

Let $g(\mathbf{K_T})$ denote the objective function in (5.8) and $\mathbf{K}_{\mathbf{T}}^{\mathbf{p}}(\tilde{B})$ be the optimal solution of (5.8). The corresponding optimal borrowing $e_T^p(\tilde{B})$ is equal to $\left(c_T\mathbf{1}'\mathbf{K}_{\mathbf{T}}^{\mathbf{p}}(\tilde{B}) + F_T - \tilde{B}\right)^+$. For $\tilde{B} > F_T$, the function $g(\mathbf{K_T})$ has a kink and is not differentiable at $\mathbf{1}'\mathbf{K_T} = \frac{\tilde{B}-F_T}{c_T}$. We rewrite (5.8) as a combination of two sub-problems $i = 0, 1$ with

$$\Psi_T(\tilde{B}) = \begin{cases} \max_i \Psi_T^i(\tilde{B}) & \text{if } \tilde{B} > F_T \\ \Psi_T^1(\tilde{B}) & \text{if } \tilde{B} \leq F_T \end{cases} \qquad (5.9)$$

such that

$$\begin{aligned}
\Psi_T^i(\tilde{B}) = \max_{\mathbf{K_T}} \quad & \tilde{B} - c_T\mathbf{1}'\mathbf{K_T} - (1-\gamma_T)F_T - a^i\left(c_T\mathbf{1}'\mathbf{K_T} + F_T - \tilde{B}\right) + \mathbf{E}\left[\mathbf{N_T}\right]'\mathbf{K_T}^{1+\frac{1}{b}} + P \\
\text{s.t.} \quad & Z_L^i \leq c_T\mathbf{1}'\mathbf{K_T} + F_T - \tilde{B} \leq Z_U^i \qquad\qquad (5.10)\\
& \mathbf{K_T} \geq \mathbf{0},
\end{aligned}$$

where $a^0 = 0, a^1 = a$ and $Z_L^0 = -\infty$, $Z_L^1 = 0$, $Z_U^0 = 0$, $Z_U^1 = E$. Subproblem 0 (1) is the restriction of the problem to the no borrowing (borrowing) regions. Let $g^i(\mathbf{K_T})$ denote the objective function and $\mathbf{K_T^{p_i}}(\tilde{B})$ be the optimal solution of sub-problem i. We have

$$g(\mathbf{K_T}) = \begin{cases} g^0(\mathbf{K_T}) & \text{if } c_T\mathbf{1}'\mathbf{K_T} + F_T \leq \tilde{B} \\ g^1(\mathbf{K_T}) & \text{if } c_T\mathbf{1}'\mathbf{K_T} + F_T > \tilde{B}. \end{cases}$$

The remainder of the proof has the following structure:

1. We show that $g^i(\mathbf{K_T})$ is strictly concave and solve each sub-problem i for $\mathbf{K_T^{p_i}}(\tilde{B})$.

2. We show that $g(\mathbf{K_T})$ is strictly concave. It follows that

$$\mathbf{K_T^p}(\tilde{B}) = \mathbf{K_T^{p_i}}(\tilde{B}) \text{ where } i = \begin{cases} arg\max_i \Psi_T^i(\tilde{B}) & \text{if } \tilde{B} > F_T \\ 1 & \text{if } \tilde{B} \leq F_T \end{cases}$$

   We derive $\Psi_T(\tilde{B})$ by using $\mathbf{K_T^{p_i}}(\tilde{B})$.

3. We compare $\Psi_T(\tilde{B})$ with $\tilde{B} - (1-\gamma_T)F_T + P$, the value of not investing in capacity, and derive $\mathbf{K_T^*}(\tilde{B})$ and $e_T^*(\tilde{B})$.

1. Solution for $\mathbf{K_T^{p_i}}(\tilde{B})$

1.a. Flexible Technology:

Let $A \doteq \mathbf{E}[\mathbf{N_F}] = \mathbf{E}\left[\left(\xi_1^{-b} + \xi_2^{-b}\right)^{-\frac{1}{b}}\right]$. The first and second order conditions in (5.10) are

$$\frac{\partial g^i}{\partial K_F} = -c_F - a^i c_F + (1+1/b)\ A\, K_F^{1/b},$$
$$\frac{\partial^2 g^i}{\partial K_F^2} = \frac{1}{b}(1+1/b)\ A\, K_F^{(1/b-1)}.$$

Since $b < -1$, we have $\lim_{K_F \to 0^+} \frac{\partial^2 K_F^{(1/b-1)}}{\partial K_F^2} \to \infty$ and $\frac{\partial^2 K_F^{(1/b-1)}}{\partial K_F^2} > 0 \ \ \forall K_F \geq 0$. With $b < -1$, it follows that $\frac{\partial^2 g^i}{\partial K_F^2} < 0$ for $K_F \geq 0$ and the function $g^i(K_F)$ is strictly concave for $i = 0, 1$. Since the constraints in (5.10) are linear, first-order KKT conditions are necessary and sufficient for optimality for each sub-problem i and $K_F^{p_i}(\tilde{B})$ is unique.

From KKT conditions if i has a non-empty feasible region then the optimal solution is either the solution of $\frac{\partial g^i}{\partial K_F} = 0$, $K_F^{p_i}(\tilde{B}) = \left(\frac{A(1+\frac{1}{b})}{c_F(1+a^i)}\right)^{-b}$, or is a boundary solution. Since $\tilde{B} > F_F$ for $i = 0$ from (5.9) and $\tilde{B} > F_F - E$ for $i = 1$ from (5.6), the non-negativity constraint is never binding in (5.10). Since $\lim_{K_F \to 0^+} \frac{\partial g^i}{\partial K_F} \to \infty$, $K_F = 0$ is never optimal. If $\frac{Z_L^i + \tilde{B} - F_F}{c_F} > 0$ and $\frac{\partial g^i}{\partial K_F} < 0$ at this point, then $K_F^{p_i}(\tilde{B}) = \frac{Z_L^i + \tilde{B} - F_F}{c_F}$, i.e., the optimal solution occurs at the lower bound of the financing constraint. If $\frac{\partial g^i}{\partial K_F} > 0$ at $K_F = \frac{Z_U^i + \tilde{B} - F_F}{c_F} > 0$, then $K_F^{p_i}(\tilde{B}) = \frac{Z_U^i + \tilde{B} - F_F}{c_F}$, i.e., the optimal solution occurs at the upper bound of the financing constraint. To summarize, $K_F^{p_i}(\tilde{B})$ for $i = 0, 1$ is characterized by

$$\mathbf{K_F^{p_0}}(\tilde{\mathbf{B}}) = \begin{cases} \mathbf{K_F^0} \doteq \left(\frac{A(1+\frac{1}{b})}{c_F}\right)^{-b} & \text{if } c_F\mathbf{K_F^0} + F_F - \tilde{B} \leq 0 \\ \overline{\mathbf{K}}_\mathbf{F} \doteq \left(\frac{\tilde{B}-F_F}{c_F}\right) & \text{if } c_F\mathbf{K_F^0} + F_F - \tilde{B} > 0, \end{cases} \tag{5.11}$$

$$\mathbf{K_F^{p_1}}(\tilde{\mathbf{B}}) = \begin{cases} \overline{\mathbf{K}}_\mathbf{F} \doteq \left(\frac{\tilde{B}-F_F}{c_F}\right) & \text{if } c_F\mathbf{K_F^1} + F_F - \tilde{B} \leq 0 \\ \mathbf{K_F^1} \doteq \left(\frac{A(1+\frac{1}{b})}{c_F(1+a)}\right)^{-b} & \text{if } 0 < c_F\mathbf{K_F^1} + F_F - \tilde{B} \leq E \\ \overline{\overline{\mathbf{K}}}_\mathbf{F} \doteq \left(\frac{E+\tilde{B}-F_F}{c_F}\right) & \text{if } c_F\mathbf{K_F^1} + F_F - \tilde{B} > E. \end{cases}$$

Here, $\mathbf{K_F^0}$ is the budget-unconstrained optimal capacity investment and $\mathbf{K_F^1}$ is the credit-unconstrained optimal capacity investment.

1.b. Dedicated Technology:

We obtain

$$\begin{aligned} \frac{\partial^2 g^i}{\partial (K_D^j)^2} &= \frac{1}{b}(1+1/b)\ \bar{\xi}_j\ (K_D^j)^{(1/b-1)} < 0, \\ \frac{\partial^2 g^i}{\partial (K_D^1)^2}\frac{\partial^2 g^i}{\partial (K_D^2)^2} - \left[\frac{\partial^2 g^i}{\partial K_D^1 K_D^2}\right]^2 &= \prod_j \frac{1}{b}(1+1/b)\ \bar{\xi}_j\ (K_D^j)^{(1/b-1)} - 0 > 0 \end{aligned}$$

for $i = 0, 1$ and $j = 1, 2$. Therefore, the Hessian matrix $D^2 g^i(\mathbf{K_D})$ is negative definite for $\mathbf{K_D} \geq \mathbf{0}$ and $g^i(\mathbf{K_D})$ is strictly concave. Since the constraints in (5.10) are linear, first-order KKT conditions are necessary and sufficient for optimality in each sub-problem i and $\mathbf{K_D^{p_i}}(\tilde{\mathbf{B}})$ is unique.

If $\mathbf{K_D^{p_i}}(\tilde{B})$ is an optimal solution to (5.10), then there exist $\boldsymbol{\lambda}^{i'} = (\lambda_1^i, \lambda_2^i)$ and $\boldsymbol{\mu}^{i'} = (\mu_1^i, \mu_2^i)$ that satisfy

$$\begin{aligned} c_D\mathbf{1}'\mathbf{K_D^{p_i}}(\tilde{B}) + F_D - \tilde{B} &\leq Z_U^i, & (5.12) \\ c_D\mathbf{1}'\mathbf{K_D^{p_i}}(\tilde{B}) + F_D - \tilde{B} &\geq Z_L^i, & (5.13) \\ \mathbf{K_D^{p_i}}(\tilde{B}) &\geq \mathbf{0}, & (5.14) \\ -(1+\boldsymbol{a}^i)c_D + (1+1/b)\ \bar{\boldsymbol{\xi}}\ \mathbf{K_D^{p_i}}(\tilde{B})^{1/b} - c_D(\lambda_1^i - \lambda_2^i) + \boldsymbol{\mu}^i &= \mathbf{0}, & (5.15) \\ \lambda_1^i[Z_U^i - c_D\mathbf{1}'\mathbf{K_D^{p_i}}(\tilde{B}) - F_D + \tilde{B}] &= 0, & (5.16) \\ \lambda_2^i[-Z_L^i + c_D\mathbf{1}'\mathbf{K_D^{p_i}}(\tilde{B}) + F_D - \tilde{B}] &= 0, & (5.17) \\ \boldsymbol{\mu}^i\mathbf{K_D^{p_i}}(\tilde{B}) &= \mathbf{0} & (5.18) \end{aligned}$$

with $\boldsymbol{\lambda}^i \geq \mathbf{0}$ and $\boldsymbol{\mu}^i \geq \mathbf{0}$ for $i = 0, 1$. Observe that $\lim_{K_D^j \to 0^+} \frac{\partial g^i}{\partial K_D^j} \to \infty$ for $j = 1, 2$, so it is never optimal to invest in only one of the resources. Since we will compare $\Psi_D(\tilde{B})$ with $\tilde{B} - (1 - \gamma_D)F_D + P$ (the value of not investing in either resource) in Step 3, we can focus on $\mathbf{K}_\mathbf{D}^{\mathbf{P_i}}(\tilde{B}) > \mathbf{0}$ here. This implies $\boldsymbol{\mu}^i = \mathbf{0}$ for (5.18) to be satisfied.

Case 1: $c_D \mathbf{1}'\mathbf{K}_\mathbf{D}^{\mathbf{P_i}}(\tilde{B}) + F_D - \tilde{B} < Z_U^i$ and $c_D \mathbf{1}'\mathbf{K}_\mathbf{D}^{\mathbf{P_i}}(\tilde{B}) + F_D - \tilde{B} > Z_L^i$

In this case $\boldsymbol{\lambda}^i = \mathbf{0}$, and (5.15) yields

$$\mathbf{K}_\mathbf{D}^{\mathbf{P_i}}(\tilde{B}) = \mathbf{K}_\mathbf{D}^\mathbf{i} \doteq \left(\frac{(1 + \frac{1}{b})}{c_D(1 + a^i)} \right)^{-b} \bar{\boldsymbol{\xi}}^{-b}.$$

For (5.12), (5.13) and (5.14) to be satisfied, and the solution $\mathbf{K}_\mathbf{D}^{\mathbf{P_i}}(\tilde{B}) = \mathbf{K}_\mathbf{D}^\mathbf{i}$ to be valid, we need $Z_L^i < c_D \mathbf{1}'\mathbf{K}_\mathbf{D}^\mathbf{i} + F_D - \tilde{B} < Z_U^i$. Here, $\mathbf{K}_\mathbf{D}^\mathbf{0}$ is the budget-unconstrained optimal capacity investment and $\mathbf{K}_\mathbf{D}^\mathbf{1}$ is the credit-unconstrained optimal capacity investment.

Case 2: $c_D \mathbf{1}'\mathbf{K}_\mathbf{D}^{\mathbf{P_i}}(\tilde{B}) + F_D - \tilde{B} = Z_U^i$

In this case (5.13) holds as a strict inequality, so $\lambda_2^i = 0$ for (5.17) to be satisfied. Rewriting the equality as $K_D^2 = \frac{Z_U^i + \tilde{B} - F_F - c_D K_D^1}{c_D}$, and combining this with (5.15) yields

$$\mathbf{K}_\mathbf{D}^{\mathbf{P_i}}(\tilde{B})' = \left(\left(\frac{Z_U^i + B - F_D}{c_D} \right) \left(\frac{\bar{\xi}_1^{-b}}{\bar{\xi}_1^{-b} + \bar{\xi}_2^{-b}} \right), \left(\frac{Z_U^i + B - F_D}{c_D} \right) \left(\frac{\bar{\xi}_2^{-b}}{\bar{\xi}_1^{-b} + \bar{\xi}_2^{-b}} \right) \right). \quad (5.19)$$

The condition $\lambda_1^i \geq 0$ should be satisfied at optimality. After some algebra, this condition implies that (5.19) is optimal if $\tilde{B} \leq c_D \mathbf{1}'\mathbf{K}_\mathbf{D}^\mathbf{i} + F_D - Z_U^i$.

Case 3: $c_D \mathbf{1}'\mathbf{K}_\mathbf{D}^{\mathbf{P_i}}(\tilde{B}) + F_D - \tilde{B} = Z_L^i$

This case is only relevant for $i = 1$ since $Z_L^0 = -\infty$. In this case, (5.12) holds as a strict inequality, so $\lambda_1^1 = 0$ for (5.16) to be satisfied. Rewriting the equality as $K_D^2 = \frac{Z_L^1 + \tilde{B} - F_F - c_D K_D^1}{c_D}$, and combining with (5.15) yields

$$\mathbf{K}_\mathbf{D}^{\mathbf{P_1}}(\tilde{B})' = \left(\left(\frac{Z_L^1 + B - F_D}{c_D} \right) \left(\frac{\bar{\xi}_1^{-b}}{\bar{\xi}_1^{-b} + \bar{\xi}_2^{-b}} \right), \left(\frac{Z_L^1 + B - F_D}{c_D} \right) \left(\frac{\bar{\xi}_2^{-b}}{\bar{\xi}_1^{-b} + \bar{\xi}_2^{-b}} \right) \right). \quad (5.20)$$

The condition $\lambda_2^1 \geq 0$ should be satisfied at optimality. After some algebra, this condition implies that (5.20) is optimal if $\tilde{B} \geq c_D \mathbf{1}'\mathbf{K}_\mathbf{D}^\mathbf{1} + F_D - Z_L^1$.

Combining cases 1, 2 and 3, $\mathbf{K_D^{P_i}}(\tilde{\mathbf{B}})$ for $i = 0, 1$ is characterized by

$$\mathbf{K_D^{P_0}}(\tilde{\mathbf{B}}) = \begin{cases} \mathbf{K_D^0} = \left(\frac{(1+\frac{1}{b})}{c_D}\right)^{-b} \bar{\boldsymbol{\xi}}^{-b} & \text{if } c_D \mathbf{1}'\mathbf{K_D^0} + F_D - \tilde{B} \leq 0 \\ \overline{\mathbf{K}}'_\mathbf{D} = \left(\left(\frac{B-F_D}{c_D}\right)\left(\frac{\bar{\xi}_1^{-b}}{\bar{\xi}_1^{-b}+\bar{\xi}_2^{-b}}\right), \left(\frac{B-F_D}{c_D}\right)\left(\frac{\bar{\xi}_2^{-b}}{\bar{\xi}_1^{-b}+\bar{\xi}_2^{-b}}\right)\right) & \text{if } c_D \mathbf{1}'\mathbf{K_D^0} + F_D - \tilde{B} > 0, \end{cases} \quad (5.21$$

$$\mathbf{K_D^{P_1}}(\tilde{\mathbf{B}}) = \begin{cases} \overline{\mathbf{K}}'_\mathbf{D} = \left(\left(\frac{B-F_D}{c_D}\right)\left(\frac{\bar{\xi}_1^{-b}}{\bar{\xi}_1^{-b}+\bar{\xi}_2^{-b}}\right), \left(\frac{B-F_D}{c_D}\right)\left(\frac{\bar{\xi}_2^{-b}}{\bar{\xi}_1^{-b}+\bar{\xi}_2^{-b}}\right)\right) & \text{if } c_D \mathbf{1}'\mathbf{K_D^1} + F_D - \tilde{B} \leq 0 \\ \mathbf{K_D^1} = \left(\frac{(1+\frac{1}{b})}{c_D(1+a)}\right)^{-b} \bar{\boldsymbol{\xi}}^{-b} & \text{if } 0 < c_D \mathbf{1}'\mathbf{K_D^1} + F_D - \tilde{B} \leq E \\ \overline{\overline{\mathbf{K}}}'_\mathbf{D} = \left(\left(\frac{E+B-F_D}{c_D}\right)\left(\frac{\bar{\xi}_1^{-b}}{\bar{\xi}_1^{-b}+\bar{\xi}_2^{-b}}\right), \left(\frac{E+B-F_D}{c_D}\right)\left(\frac{\bar{\xi}_2^{-b}}{\bar{\xi}_1^{-b}+\bar{\xi}_2^{-b}}\right)\right) & \text{if } c_D \mathbf{1}'\mathbf{K_D^1} + F_D - \tilde{B} > E. \end{cases}$$

2. Solution for $\mathbf{K_T^P}(\tilde{\mathbf{B}})$ and $\Psi_T(\tilde{B})$:

To show that $g(\mathbf{K_T})$ is strictly concave, we need to show that $\forall\, \mathbf{K_T^I}, \mathbf{K_T^{II}} \geq 0$ and $\lambda \in (0,1)$,

$$g(\lambda \mathbf{K_T^I} + (1-\lambda)\mathbf{K_T^{II}}) - \lambda g(\mathbf{K_T^I}) - (1-\lambda) g(\mathbf{K_T^{II}}) > 0. \qquad (5.22)$$

Since $g^i(\mathbf{K_T})$ is strictly concave, we only need to focus on $\mathbf{K_T^I}, \mathbf{K_T^{II}}$ such that $c_T\mathbf{1}'\mathbf{K_T^I} + F_T \leq \tilde{B}$ and $c_T\mathbf{1}'\mathbf{K_T^{II}} + F_T > \tilde{B}$. We have two cases to consider. First, if $c_T\mathbf{1}'\left(\lambda \mathbf{K_T^I} + (1-\lambda)\mathbf{K_T^{II}}\right) + F_T \leq \tilde{B}$ then after some algebra, the left-hand side of (5.22) becomes

$$\mathbf{E}[\mathbf{N_T}]'(\lambda \mathbf{K_T^I} + (1-\lambda)\mathbf{K_T^{II}})^{1+\frac{1}{b}} - \lambda \mathbf{E}[\mathbf{N_T}]'\mathbf{K_T^I}^{1+\frac{1}{b}} - (1-\lambda)\mathbf{E}[\mathbf{N_T}]'\mathbf{K_T^{II}}^{1+\frac{1}{b}} + (1-\lambda)a(c_T\mathbf{1}'\mathbf{K_T^{II}} + F_T - \tilde{B})$$

Since $x^{1+\frac{1}{b}}$ is strictly concave for $x \geq 0$ and $c_T\mathbf{1}'\mathbf{K_T^{II}} + F_T - \tilde{B}$ is positive by definition, the above equation is strictly greater than 0. Second, if $c_T\mathbf{1}'\left(\lambda \mathbf{K_T^I} + (1-\lambda)\mathbf{K_T^{II}}\right) + F_T > \tilde{B}$ then after some algebra, the left-hand side of (5.22) becomes

$$\mathbf{E}[\mathbf{N_T}]'(\lambda \mathbf{K_T^I} + (1-\lambda)\mathbf{K_T^{II}})^{1+\frac{1}{b}} - \lambda \mathbf{E}[\mathbf{N_T}]'\mathbf{K_T^I}^{1+\frac{1}{b}} - (1-\lambda)\mathbf{E}[\mathbf{N_T}]'\mathbf{K_T^{II}}^{1+\frac{1}{b}} - \lambda a_T(c_T\mathbf{1}'\mathbf{K_T^I} + F_T - \tilde{B}).$$

Since $x^{1+\frac{1}{b}}$ is strictly concave for $x \geq 0$ and $c_T\mathbf{1}'\mathbf{K_T^I} + F_T - \tilde{B}$ is negative by definition, the equation above is strictly greater than 0. Since (5.22) is satisfied for both cases, $g(\mathbf{K_T})$ is strictly concave. It follows that

$$\mathbf{K_T^P}(\tilde{B}) = \mathbf{K_T^{P_i}}(\tilde{B}) \text{ where } i = \begin{cases} arg\max_i \Psi_T^i(\tilde{B}) & \text{if } \tilde{B} > F_T \\ 1 & \text{if } \tilde{B} \leq F_T \end{cases}$$

is the unique maximizer of g. Combining (5.11) and (5.21), the unique optimal solution to problem (5.8) and the corresponding optimal amount of borrowing are given by

$$\mathbf{K_T^p}(\tilde{B}) = \begin{cases} \mathbf{K_T^0} & \text{if } c_T\mathbf{1}'\mathbf{K_T^0} + F_T \le \tilde{B} \\ \overline{\mathbf{K}}_\mathbf{T} & \text{if } c_T\mathbf{1}'\mathbf{K_T^1} + F_T \le \tilde{B} < c_T\mathbf{1}'\mathbf{K_T^0} + F_T \\ \mathbf{K_T^1} & \text{if } c_T\mathbf{1}'\mathbf{K_T^1} + F_T - E \le \tilde{B} < c_T\mathbf{1}'\mathbf{K_T^1} + F_T \\ \overline{\overline{\mathbf{K}}}_\mathbf{T} & \text{if } \tilde{B} < c_T\mathbf{1}'\mathbf{K_T^1} + F_T - E, \end{cases} \tag{5.23}$$

$$e_T^p(\tilde{B}) = \left(c_T\mathbf{1}'\mathbf{K_T^p}(\tilde{B}) + F_T - \tilde{B}\right)^+$$

where

$$\mathbf{K_D^0}' = \left(\left(\frac{\bar{\xi}_1\left(1+\frac{1}{b}\right)}{c_D}\right)^{-b}, \left(\frac{\bar{\xi}_2\left(1+\frac{1}{b}\right)}{c_D}\right)^{-b}\right)$$

$$\overline{\mathbf{K}}_\mathbf{D}' = \left(\left(\frac{B-F_D}{c_D}\right)\left(\frac{\bar{\xi}_1^{-b}}{\bar{\xi}_1^{-b}+\bar{\xi}_2^{-b}}\right), \left(\frac{B-F_D}{c_D}\right)\left(\frac{\bar{\xi}_2^{-b}}{\bar{\xi}_1^{-b}+\bar{\xi}_2^{-b}}\right)\right)$$

$$\mathbf{K_D^1}' = \left(\left(\frac{\bar{\xi}_1\left(1+\frac{1}{b}\right)}{c_D(1+a)}\right)^{-b}, \left(\frac{\bar{\xi}_2\left(1+\frac{1}{b}\right)}{c_D(1+a)}\right)^{-b}\right)$$

$$\overline{\overline{\mathbf{K}}}_\mathbf{D}' = \left(\left(\frac{E+B-F_D}{c_D}\right)\left(\frac{\bar{\xi}_1^{-b}}{\bar{\xi}_1^{-b}+\bar{\xi}_2^{-b}}\right), \left(\frac{E+B-F_D}{c_D}\right)\left(\frac{\bar{\xi}_2^{-b}}{\bar{\xi}_1^{-b}+\bar{\xi}_2^{-b}}\right)\right)$$

$$K_F^0 = \left(\frac{A\left(1+\frac{1}{b}\right)}{c_F}\right)^{-b}$$

$$\overline{K}_F = \left(\frac{B-F_F}{c_F}\right)$$

$$K_F^1 = \left(\frac{A\left(1+\frac{1}{b}\right)}{c_F(1+a)}\right)^{-b}$$

$$\overline{\overline{K}}_F = \left(\frac{E+B-F_F}{c_F}\right).$$

We substitute (5.23) in (5.8) and find

$$\Psi_T(\tilde{B}) = \begin{cases} \tilde{B} - (1-\gamma_T)F_T + \frac{c_T\mathbf{1}'\mathbf{K_T^0}}{-(b+1)} + P & \text{if } c_T\mathbf{1}'\mathbf{K_T^0} + F_T \le \tilde{B} \\ M_T\left(\frac{\tilde{B}-F_T}{c_T}\right)^{1+\frac{1}{b}} + \gamma_T F_T + P & \text{if } c_T\mathbf{1}'\mathbf{K_T^1} + F_T \le \tilde{B} < c_T\mathbf{1}'\mathbf{K_T^0} + F_T \\ (\tilde{B}-F_T)(1+a) + \frac{c_T\mathbf{1}'\mathbf{K_T^1}(1+a)}{-(b+1)} + \gamma_T F_T + P & \text{if } c_T\mathbf{1}'\mathbf{K_T^1} + F_T - E \le \tilde{B} < c_T\mathbf{1}'\mathbf{K_T^1} + F_T \\ -E(1+a) + M_T\left(\frac{E+\tilde{B}-F_T}{c_T}\right)^{1+\frac{1}{b}} + \gamma_T F_T + P & \text{if } \tilde{B} < c_T\mathbf{1}'\mathbf{K_T^1} + F_T - E. \end{cases} \tag{5.24}$$

where $M_F = \mathbf{E}\left[\left(\xi_1^{-b}+\xi_2^{-b}\right)^{-\frac{1}{b}}\right]$ and $M_D = \left(\bar{\xi}_1^{-b}+\bar{\xi}_2^{-b}\right)^{-\frac{1}{b}}$. It follows from (5.6) that $\Psi_T(\tilde{B})$ is relevant (and is defined) only for $\tilde{B} > F_T - E$.

3. Solution for $\mathbf{K_T^*}(\tilde{B})$ and $e_T^*(\tilde{B})$:

To complete the characterization of $\mathbf{K}^*_\mathbf{T}(\tilde{B})$ and $e^*_T(\tilde{B})$, we compare $\Psi_T(\tilde{B})$ with $\tilde{B}-(1-\gamma_T)F_T+P$ (the value of the not borrowing and not investing in capacity) for $\tilde{B}>F_T-E$ and establish that the two functions intersect at most once on $\tilde{B}\in(F_T-E,\infty)$; and find $\mathbf{K}^*_\mathbf{T}(\tilde{B})$ and $e^*_T(\tilde{B})$.

For $\tilde{B}>F_T-E$, we define $G_T(\tilde{B})\doteq\Psi_T(\tilde{B})-(\tilde{B}-(1-\gamma_T)F_T+P)$, the difference between the equity values in (5.24) and not borrowing and not investing in capacity. It is easy to verify that, $\lim_{\tilde{B}\to\tilde{B}_k^+}\Psi_T(\tilde{B})=\lim_{\tilde{B}\to\tilde{B}_k^-}\Psi_T(\tilde{B})$ for $\forall\ \tilde{B}_k>F_T-E$ therefore, $\Psi_T(\tilde{B})$ and, in turn, $G_T(\tilde{B})$ are continuous functions of $\tilde{B}$. We have

$$\frac{\partial G_T(\tilde{B})}{\partial\tilde{B}}=\begin{cases}0 & \text{if } c_T\mathbf{1}'\mathbf{K^0_T}+F_T\le\tilde{B}\\ \frac{M_T}{c_T}(1+\frac{1}{b})\left(\frac{\tilde{B}-F_T}{c_T}\right)^{\frac{1}{b}}-1 & \text{if } c_T\mathbf{1}'\mathbf{K^1_T}+F_T\le\tilde{B}<c_T\mathbf{1}'\mathbf{K^0_T}+F_T\\ a & \text{if } c_T\mathbf{1}'\mathbf{K^1_T}+F_T-E\le\tilde{B}<c_T\mathbf{1}'\mathbf{K^1_T}+F_T\\ \frac{M_T}{c_T}(1+\frac{1}{b})\left(\frac{E+\tilde{B}-F_T}{c_T}\right)^{\frac{1}{b}}-1 & \text{if } \tilde{B}<c_T\mathbf{1}'\mathbf{K^1_T}+F_T-E.\end{cases}\tag{5.25}$$

For $c_T\mathbf{1}'\mathbf{K^1_T}+F_T\le\tilde{B}<c_T\mathbf{1}'\mathbf{K^0_T}+F_T$,

$$\frac{M_T}{c_T}(1+\frac{1}{b})\left(\frac{\tilde{B}-F_T}{c_T}\right)^{\frac{1}{b}}-1>\frac{M_T}{c_T}(1+\frac{1}{b})\ (\mathbf{1}'\mathbf{K^0_T})^{\frac{1}{b}}-1=0,\tag{5.26}$$

and for $\tilde{B}<c_T\mathbf{1}'\mathbf{K^1_T}+F_T-E$,

$$\frac{M_T}{c_T}(1+\frac{1}{b})\left(\frac{E+\tilde{B}-F_T}{c_T}\right)^{\frac{1}{b}}-1>\frac{M_T}{c_T}(1+\frac{1}{b})\ (\mathbf{1}'\mathbf{K^1_T})^{\frac{1}{b}}-1=a.\tag{5.27}$$

It follows that $\lim_{\tilde{B}\to\tilde{B}_k^+}\frac{\partial}{\partial\tilde{B}}G_T(\tilde{B})=\lim_{\tilde{B}\to\tilde{B}_k^-}\frac{\partial}{\partial\tilde{B}}G_T(\tilde{B})$ on the domain of $G_T(.)$. Therefore $G_T(\tilde{B})$ is differentiable for $\tilde{B}>F_T-E$ and $\frac{\partial}{\partial\tilde{B}}G_T(\tilde{B})\ge0$ with equality holding only for $\tilde{B}\ge c_T\mathbf{1}'\mathbf{K^0_T}+F_T$. For $c_T\mathbf{1}'\mathbf{K^0_T}+F_T\le\tilde{B}$,

$$G_T(\tilde{B})=\tilde{B}-(1-\gamma_T)F_T+\frac{c_T\mathbf{1}'\mathbf{K^0_T}}{-(b+1)}+P-(\tilde{B}-(1-\gamma_T)F_T+P)=\frac{c_T\mathbf{1}'\mathbf{K^0_T}}{-(b+1)}>0\tag{5.28}$$

We showed that $G_T(\tilde{B})$ strictly increases for $\tilde{B}\in(F_T-E,c_T\mathbf{1}'\mathbf{K^0_T}+F_T)$ and is positive for $\tilde{B}\in[c_T\mathbf{1}'\mathbf{K^0_T}+F_T,\infty)$. Let $\widehat{B}_T$ denote the budget level at which the two equity value curves intersect, i.e. $G_T(\widehat{B}_T)=0$. For $F_T\ge E$, we have $\lim_{\tilde{B}\to(F_T-E)^+}G_T(\tilde{B})=-aE<0$. Since $G_T(\tilde{B})$ strictly increases in $\tilde{B}$, it follows that for $F_T\ge E$, there exists a unique $\widehat{B}_T>F_T-E$ such that $G_T(\widehat{B}_T)=0$. For $F_T<E$, the domain of $G_T(\tilde{B})$ is $[0,\infty)$. For notational convenience, we let $\widehat{B}_T\doteq0$ if the two curves do not intersect on this domain ($G_T(\tilde{B})>0$ for $\tilde{B}\ge0$). Since $G_T(\tilde{B})$ strictly increases in $\tilde{B}$, it follows that for $F_T<E$,

$\widehat{B}_T$, if it exists on $[0,\infty)$, is unique. For $\tilde{B} \le \widehat{B}_T$ we have $\mathbf{K}^*_\mathbf{T}(\tilde{B}) = \mathbf{0}$ and $e^*_T(\tilde{B}) = 0$. Combining this with (5.23) gives the desired result. ■

Proof of Corollary 1: The expected (stage 1) equity value of the firm with a given budget level $\tilde{B}$ follows directly from Proposition 2:

$$\pi_T(\tilde{B}) = \begin{cases} \tilde{B} - (1-\gamma_T)F_T + \frac{c_T \mathbf{1}'\mathbf{K}^0_\mathbf{T}}{-(b+1)} + P & \text{if } \tilde{B} \in \Omega^0_T \\ M_T\left(\frac{\tilde{B}-F_T}{c_T}\right)^{1+\frac{1}{b}} + \gamma_T F_T + P & \text{if } \tilde{B} \in \Omega^1_T \\ (\tilde{B}-F_T)(1+a) + \frac{c_T\mathbf{1}'\mathbf{K}^1_\mathbf{T}(1+a)}{-(b+1)} + \gamma_T F_T + P & \text{if } \tilde{B} \in \Omega^2_T \\ -E(1+a) + M_T\left(\frac{E+\tilde{B}-F_T}{c_T}\right)^{1+\frac{1}{b}} + \gamma_T F_T + P & \text{if } \tilde{B} \in \Omega^3_T \\ \tilde{B} - (1-\gamma_T)F_T + P & \text{if } \tilde{B} \in \Omega^4_T \end{cases} \tag{5.29}$$

where $M_F = \mathbf{E}\left[\left(\xi_1^{-b} + \xi_2^{-b}\right)^{-\frac{1}{b}}\right]$ and $M_D = \left(\overline{\xi}_1^{-b} + \overline{\xi}_2^{-b}\right)^{-\frac{1}{b}}$.

We calculate

$$\left(\frac{\partial \pi_T(\tilde{B})}{\partial \tilde{B}}, \frac{\partial^2 \pi_T(\tilde{B})}{\partial \tilde{B}^2}\right) = \begin{cases} (1,0) & \text{if } \tilde{B} \in \Omega^0_T \\ \left(\frac{M_T}{c_T}(1+1/b)\left(\frac{\tilde{B}-F_T}{c_T}\right)^{\frac{1}{b}}, \frac{1}{b}\frac{M_T}{c_T^{(1+1/b)}}(1+1/b)(\tilde{B}-F_T)^{\frac{1}{b}-1}\right) & \text{if } \tilde{B} \in \Omega^1_T \\ (1+a,0) & \text{if } \tilde{B} \in \Omega^2_T \\ \left(\frac{M_T}{c_T}(1+1/b)\left(\frac{E+\tilde{B}-F_T}{c_T}\right)^{\frac{1}{b}}, \frac{1}{b}\frac{M_T}{c_T^{(1+1/b)}}(1+1/b)(E+\tilde{B}-F_T)^{\frac{1}{b}-1}\right) & \text{if } \tilde{B} \in \Omega^3_T \\ (1,0) & \text{if } \tilde{B} \in \Omega^4_T \end{cases}$$

at the points where $\pi_T(\tilde{B})$ is differentiable. It is easy to verify that $\lim_{\tilde{B}\to\tilde{B}_k^+} \frac{\partial}{\partial \tilde{B}}\pi_T(\tilde{B}) = \lim_{\tilde{B}\to\tilde{B}_k^-} \frac{\partial}{\partial \tilde{B}}\pi_T(\tilde{B})$ for $\tilde{B}_k \in \Omega^{0123}_T$, and $\pi_T(\tilde{B})$ is differentiable everywhere in its domain except at $\widehat{B}_T$. Since $\pi_T(\tilde{B})$ is a continuous function of $\tilde{B}$ it follows that $\pi_T(\tilde{B})$ is strictly increasing in $\tilde{B}$.

We have $\frac{\partial^2}{\partial \tilde{B}^2}\pi_T(\tilde{B}) \le 0$ for each Ω^i_T and $\pi_T(\tilde{B})$ is piecewise concave. From (5.26) we obtain $\frac{\partial}{\partial \tilde{B}}\pi_T(\tilde{B}) > 1$ for $\tilde{B} \in \Omega^1_T$ and from (5.27) we have $\frac{\partial}{\partial \tilde{B}}\pi_T(\tilde{B}) > 1+a$ for $\tilde{B} \in \Omega^3_T$. Since $\pi_T(\tilde{B})$ is only kinked at $\widehat{B}_T$ it follows that $\pi_T(\tilde{B})$ is concave in $\tilde{B}$ for $\tilde{B} \ge \widehat{B}_T$, but not globally concave. ■

Proof of Proposition 3: The optimal risk management level H^*_T is given by

$$\begin{aligned} H^*_T = argmax_{H_T} \quad & \mathbf{E}\left[\pi_T(B_{FRM}(\alpha_1, H_T))\right] \\ \text{s.t.} \quad & -\frac{\omega_0^{FRM}}{\overline{\alpha}_1} \le H_T \le \omega_1^{FRM} \end{aligned} \tag{5.30}$$

Since $\boldsymbol{\xi}$ and α_1 are independent,

$$\mathbf{E}_{\boldsymbol{\xi},\alpha_1}\left[\pi_T(B_{FRM}(\alpha_1, H_T))\right] = \mathbf{E}_{\alpha_1}\left[\mathbf{E}_{\boldsymbol{\xi}}\left[\pi_T(B_{FRM}(\alpha_1, H_T))\right]\right] = \mathbf{E}_{\alpha_1}\left[\pi_T(B_{FRM}(\alpha_1, H_T))\right].$$

Therefore we can write the expectation in (5.30) over α_1. Let $r_{\alpha_1}(.)$ and $R_{\alpha_1}(.)$ denote the density and distribution function of α_1, respectively. Since $B_{FRM}(\alpha_1, H_T) = \omega_0^{FRM} + \alpha_1(\omega_1^{FRM} - H_T) + \overline{\alpha}_1 H_T$, for each H_T the unique distribution function of $B_{FRM}(H_T)$ is

$$R_{B_{FRM}(H_T)}(\tilde{B}) = R_{\alpha_1}\left(\frac{\tilde{B} - \omega_0^{FRM} - \overline{\alpha}_1 H_T}{\omega_1^{FRM} - H_T}\right) \qquad \tilde{B} \geq \omega_0^{FRM} + \overline{\alpha}_1 H_T. \tag{5.31}$$

It follows that H_T determines the range and the probability distribution of the available budget in stage 1. Since we do not impose any specific assumption on the type of the distribution of α_1, we will use general structural properties of the optimization problem (5.30) to solve for H_T^*. In particular, we will focus on the functional form of $\pi_T(\tilde{B})$ since the expected (stage 0) value of the equity is the expectation of this function with respect to the budget random variable. We first provide the following lemma that we will use throughout the proof. The proof is relegated to Appendix 5.

Lemma 1 *There exist unique fixed cost threshold $\overline{F}_T$ such that $\widehat{B}_T = 0$ iff $F_T \leq \overline{F}_T$, and $\widehat{B}_T > 0$ iff $F_T > \overline{F}_T$.*

We now conclude the proof by analyzing each case in Proposition 3.

Case (i), $F_T \leq \overline{F}_T$:

It follows from Lemma 1 that $\widehat{B}_T = 0$. Since $\widehat{B}_T = 0$, from Corollary 1 we have that $\pi_T(\tilde{B})$ is concave for $\tilde{B} \geq 0$. From Jensen's inequality,

$$\begin{aligned} \mathbf{E}\left[\pi_T(B_{FRM}(\alpha_1, H_T))\right] \leq \pi_T\left(\mathbf{E}[B_{FRM}(\alpha_1, H_T)]\right) &= \pi_T\left(\omega_0^{FRM} + \overline{\alpha}_1 \omega_1^{FRM}\right)\right) \\ &= \pi_T\left(B_{FRM}\left(\alpha_1, \omega_1^{FRM}\right)\right) \end{aligned} \tag{5.32}$$

for $H_T \in \left[-\frac{\omega_0^{FRM}}{\overline{\alpha}_1}, \omega_1^{FRM}\right]$. This implies that $H_T^* = \omega_1^{FRM}$.

Case (ii), $F_T > \overline{F}_T$:

From Lemma 1, we have $\widehat{B_T} > 0$ and we cannot guarantee the concavity of π_T for the whole range of $\tilde{B}$. Therefore Jensen's inequality is not sufficient to find H_T^*. In this case, H_T^* is either a solution to the first order condition $\frac{\partial}{\partial H_T}\mathbf{E}[\pi_T] = 0$, or occurs at a boundary, i.e. $H_T^* \in \{-\frac{\omega_0^{FRM}}{\overline{\alpha}_1}, \omega_1^{FRM}\}$. To write the first-order condition, we utilize the following lemma proven in Appendix 5:

Lemma 2 *For any argument κ_T of π_T, the expectation and the derivative operators can be interchanged, i.e. $\frac{\partial}{\partial \kappa_T}\mathbf{E}[\pi_T] = \mathbf{E}\left[\frac{\partial}{\partial \kappa_T}\pi_T\right]$.*

Let

$\alpha_T^0 \doteq \frac{c_T \mathbf{1}'\mathbf{K}_T^0 + F_T - \omega_0^{FRM} - H_T\overline{\alpha}_1}{\omega_1^{FRM} - H_T}, \qquad \alpha_T^1 \doteq \frac{c_T \mathbf{1}'\mathbf{K}_T^1 + F_T - \omega_0^{FRM} - H_T\overline{\alpha}_1}{\omega_1^{FRM} - H_T},$

$\alpha_T^2 \doteq \frac{c_T \mathbf{1}'\mathbf{K}_T^1 + F_T - E - \omega_0^{FRM} - H_T\overline{\alpha}_1}{\omega_1^{FRM} - H_T}, \quad \alpha_T^B \doteq \frac{\widehat{B}_T - \omega_0^{FRM} - H_T\overline{\alpha}_1}{\omega_1^{FRM} - H_T}.$

From Lemma 2 (letting $\kappa_T = H_T$), we can write the first-order condition $\frac{\partial}{\partial H_T}\mathbf{E}[\pi_T]$ by using the expression for $\pi_T(\tilde{B})$ in (5.29) of Corollary 1 and the equivalence in (5.31). The integration ranges correspond to the regions Ω_T^i in (5.29) of Corollary 1.

$$\begin{aligned}
\mathbf{E}\left[\frac{\partial \pi_T}{\partial H_T}\right] &= \int_{\max(\alpha_T^0,0)}^{\infty} (\overline{\alpha}_1 - x)\, r_{\alpha_1}(x)\, dx \qquad (5.33)\\
&+ \int_{\max(\alpha_T^1,0)}^{\max(\alpha_T^0,0)} \frac{M_T}{c_T}(1+\frac{1}{b})\left(\frac{\omega_0^{FRM} + x(\omega_1^{FRM} - H_T) + \overline{\alpha}_1 H_T - F_T}{c_T}\right)^{\frac{1}{b}} (\overline{\alpha}_1 - x)\, r_{\alpha_1}(x)\, dx\\
&+ \int_{\max(\alpha_T^2,0,\alpha_T^B)}^{\max(\alpha_T^1,0)} (\overline{\alpha}_1 - x)(1+a)\, r_{\alpha_1}(x)\, dx\\
&+ \int_{\max(0,\alpha_T^B)}^{\max(\alpha_T^2,0,\alpha_T^B)} \frac{M_T}{c_T}(1+\frac{1}{b})\left(\frac{\omega_0^{FRM} + x(\omega_1^{FRM} - H_T) + \overline{\alpha}_1 H_T + E - F_T}{c_T}\right)^{\frac{1}{b}} (\overline{\alpha}_1 - x)\, r_{\alpha_1}(x)\, dx\\
&+ \int_0^{\max(0,\alpha_T^B)} (\overline{\alpha}_1 - x)\, r_{\alpha_1}(x)\, dx
\end{aligned}$$

Both the limits of integration and the integrants in (5.33) are functions of H_T. Since we do not impose any distributional assumptions on α_1 it is not always possible to find a closed-form solution for H_T^*.

We have $\alpha_T^0 > \alpha_T^1 > \alpha_T^2$ by definition. For $\omega_0^{FRM} + \overline{\alpha}_1\omega_1^{FRM} \leq \widehat{B}_T$, $\alpha_T^B \geq \overline{\alpha}_1$. Therefore, for $\omega_0^{FRM} + \overline{\alpha}_1\omega_1^{FRM} \leq \widehat{B}_T$, we either have $\alpha_T^0 > \alpha_T^1 > \alpha_t^2 > \alpha_T^B \geq \overline{\alpha}_1 > 0$ or $\alpha_T^0 > \alpha_T^1 > \alpha_T^B \geq \overline{\alpha}_1 > 0 > \alpha_T^2$. Similar to (5.26) and (5.27) we establish

$$\begin{aligned}
\int_{\alpha_T^1}^{\alpha_T^0} \frac{M_T}{c_T}(1+\frac{1}{b})\left(\frac{\omega_0^{FRM} + x(\omega_1^{FRM} - H_T) + \overline{\alpha}_1 H_T - F_T}{c_T}\right)^{\frac{1}{b}} (\overline{\alpha}_1 - x)\, r_{\alpha_1}(x)\, dx&\\
< \int_{\alpha_T^1}^{\alpha_T^0} (\overline{\alpha}_1 - x)\, r_{\alpha_1}(x)\, dx&,
\end{aligned}$$

$$\begin{aligned}
\int_{\alpha_T^B}^{\max(\alpha_T^2,\alpha_T^B)} \frac{M_T}{c_T}(1+\frac{1}{b})\left(\frac{\omega_0^{FRM} + x(\omega_1^{FRM} - H_T) + \overline{\alpha}_1 H_T - F_T}{c_T}\right)^{\frac{1}{b}} (\overline{\alpha}_1 - x)\, r_{\alpha_1}(x)\, dx&\\
< \int_{\alpha_T^B}^{\max(\alpha_T^2,\alpha_T^B)} (\overline{\alpha}_1 - x)(1+a)\, r_{\alpha_1}(x)\, dx&.
\end{aligned}$$

It follows that

$$\frac{\partial \pi_T}{\partial H_T} < \int_0^{\infty} (\overline{\alpha}_1 - x)\, r_{\alpha_1}(x)\, dx + a\int_{\alpha_T^B}^{\alpha_T^1} (\overline{\alpha}_1 - x)\, r_{\alpha_1}(x)\, dx. \qquad (5.34)$$

The first term is equal to 0 and the second term is negative, therefore $\frac{\partial \pi_T}{\partial H_T} < 0$ and $H_T^* = -\frac{\omega_0^{FRM}}{\overline{\alpha}_1}$. This concludes the proof for part (1) of this case.

If $\omega_0^{FRM} + \overline{\alpha}_1 \omega_1^{FRM} > \widehat{B}_T$, then H_T^* either satisfies $\mathbf{E}\left[\frac{\partial \pi_T}{\partial H_T}\right]\Big|_{H_T^*} = 0$ or occurs at a boundary $\{-\frac{\omega_0^{FRM}}{\overline{\alpha}_1}, \omega_1^{FRM}\}$ depending on the distributions of α_1 and $\boldsymbol{\xi}$. From Jensen's inequality, ω_1^{FRM} dominates $H_T \geq \frac{\widehat{B}_T - \omega_0^{FRM}}{\overline{\alpha}_1}$ because by (5.31) and Corollary 1, $\pi_T(B_{FRM}(\alpha_1, H_T))$ is concave over its domain for $H_T \geq \frac{\widehat{B}_T - \omega_0^{FRM}}{\overline{\alpha}_1}$. It follows that $H_T^* \in \left\{\left\{H_T < \frac{\widehat{B}_T - \omega_0^{FRM}}{\overline{\alpha}_1}\right\} \cup \{\omega_1^{FRM}\}\right\}$.

■

Proof of Proposition 4: We first prove the existence of $\overline{c}_F(c_D, \mathbf{H}^*)$. Notice from (5.30) that the optimal financial risk management level H_T^* depends on c_T[1]. For each financial risk management level H_T, the expected (stage 0) equity value $\mathbf{E}\left[\pi_T\left(c_T, B_{FRM}(\alpha_1, H_T)\right)\right]$ is a continuous function of c_T. It follows that the expected (stage 0) equity value at the optimal risk management level $\mathbf{E}\left[\pi_T\left(c_T, B_{FRM}(\alpha_1, H_T^*(c_T))\right)\right]$ is also a continuous function of c_T (because it is the upper envelope of continuous functions). For a finite $c_D > 0$, $\mathbf{E}\left[\pi_D\left(c_D, B_{FRM}(\alpha_1, H_D^*(c_D))\right)\right]$ is also finite. It is easy to prove that

$$\begin{aligned}
\lim_{c_F \to \infty} \mathbf{E}\left[\pi_F\left(c_F, B_{FRM}(\alpha_1, H_F^*(c_F))\right)\right] &= \omega_0^{FRM} + \overline{\alpha}_1 \omega_1^{FRM} - (1-\gamma_F)F_F + P, \\
\lim_{c_F \to 0} \mathbf{E}\left[\pi_F\left(c_F, B_{FRM}(\alpha_1, H_F^*(c_F))\right)\right] &\to \infty.
\end{aligned}$$

Since the equity value is continuous in c_F, if $\mathbf{E}\left[\pi_D\left(c_D, B_{FRM}(\alpha_1, H_D^*(c_D))\right)\right] > \omega_0^{FRM} + \overline{\alpha}_1 \omega_1^{FRM} - (1-\gamma_F)F_F + P$, then there exists a c_F such that the equity values with both technologies coincide. If $\mathbf{E}\left[\pi_D\left(c_D, B_{FRM}(\alpha_1, H_D^*(c_D))\right)\right] \leq \omega_0^{FRM} + \overline{\alpha}_1 \omega_1^{FRM} - (1-\gamma_F)F_F + P$ then the threshold does not exist and the flexible technology is always preferred over the dedicated technology. This concludes the proof for existence of $\overline{c}_F(c_D, \mathbf{H}^*)$. The existence of $\overline{c}_F(c_D, \mathbf{0})$ can be proven in the same manner by substituting $B_{FRM}(.)$ with $B_{-FRM}(.)$ and $H_T^*(c_T)$ with 0.

To prove the uniqueness of $\overline{c}_F(c_D, \mathbf{H}^*)$ and $\overline{c}_F(c_D, \mathbf{0})$ we first provide the following lemma and relegate the proof to Appendix 5:

Lemma 3 *In the optimal set of financial risk management levels, for a fixed level of H, the expected (stage 0) value of the equity with technology T strictly decreases in the unit capacity investment cost* ($\frac{\partial}{\partial c_T}\mathbf{E}[\pi_T(c_T, B_{FRM}(\alpha_1, H))] < 0$).

From Lemma 3 it follows that the expected (stage 0) equity value with flexible technology is

[1] Since from Proposition 3 we cannot guarantee the uniqueness of H_T^*, $H_T^*(c_T)$ is a correspondence.

strictly decreasing in c_F for any (relevant) financial risk management level H_F. This implies the uniqueness of $\bar{c}_F(c_D, \mathbf{H}^*)$. The uniqueness of $\bar{c}_F(c_D, \mathbf{0})$ follows from Lemma 3 using the identity $B_{-FRM}(\alpha_1) = B_{FRM}(\alpha_1, H)$ for $H = 0$ and $F_{FRM} = 0$. For the comparative statics results with respect to demand variability and correlation we first provide the following two lemmas and relegate their proofs to Appendix 5. Recall from Corollary 1 that $M_F(\boldsymbol{\xi}) = \mathbf{E}\left[\left(\xi_1^{-b} + \xi_2^{-b}\right)^{-\frac{1}{b}}\right]$.

Lemma 4 *$M_F(\boldsymbol{\xi}) \leq M_F(\boldsymbol{\xi})$ for $\boldsymbol{\xi}$ that is obtained from $\boldsymbol{\xi}$ with an increase in σ in one of the following ways:*

i) $\boldsymbol{\xi}$ is obtained by an increase in σ where $\boldsymbol{\xi}$ has a symmetric bivariate lognormal distribution,

ii) $\boldsymbol{\xi}$ and $\boldsymbol{\xi}$ have independent marginal distributions, equal means ($\bar{\boldsymbol{\xi}} = \overline{\boldsymbol{\xi}}$), and $\xi_i' \succeq_v \xi_i$ (ξ_i' is stochastically more variable than ξ_i) for $i = 1, 2$ or the variability ordering holds for only one of the marginals and the other marginal is identical,

iii) $\boldsymbol{\xi}$ is random ($\sigma \neq 0$) while $\boldsymbol{\xi}$ is deterministic ($\sigma = 0$).

Lemma 5 *$M_F(\boldsymbol{\xi}) \geq M_F(\boldsymbol{\xi})$ for $\boldsymbol{\xi}$ that is obtained from $\boldsymbol{\xi}$ with an increase in ρ in one of the following ways:*

i) $\boldsymbol{\xi}$ is obtained by an increase in ρ where $\boldsymbol{\xi}$ has a symmetric bivariate lognormal distribution,

ii) $\boldsymbol{\xi}$ dominates $\boldsymbol{\xi}$ according to the concordance ordering ($\boldsymbol{\xi} \succeq_c \boldsymbol{\xi}$),

iii) $\boldsymbol{\xi}$ is perfectly positively correlated ($\rho = 1$) and $\boldsymbol{\xi}$ is less than perfectly positively correlated ($\rho < 1$).

In Lemma 4 and Lemma 5, case *i* imposes distributional assumptions on $\boldsymbol{\xi}$ to analyze the effect of σ and ρ, respectively. Case *ii* of each lemma analyzes different stochastic orderings to capture the effect of product market conditions. Variability ordering is often used in the literature to analyze the effect of increasing variability. Concordance ordering $\boldsymbol{\xi} \succeq_c \boldsymbol{\xi}$, as stated in Corbett and Rajaram (2005, p. 13), essentially means that (ξ_1', ξ_2') move together more closely than (ξ_1, ξ_2). Case *iii* focuses on limiting cases.

To establish the comparative statics results, we provide the following lemma and relegate the proof to Appendix 5:

Lemma 6 *In the optimal set of financial risk management levels, for a fixed level of H, the expected (stage 0) value of the equity with technology T*

i) strictly decreases in the fixed cost of technology and strictly increases in the salvage rate ($\frac{\partial}{\partial F_T}\mathbf{E}[\pi_T(F_T, B_{FRM}(\alpha_1, H))] < 0$ *and* $\frac{\partial}{\partial \gamma_T}\mathbf{E}[\pi_T(\gamma_T, B_{FRM}(\alpha_1, H))] > 0$),

ii) decreases in unit financing cost ($\frac{\partial}{\partial a}\mathbf{E}[\pi_T(a, B_{FRM}(\alpha_1, H))] \leq 0$), *and the equality only holds for H such that* $\omega_0^{FRM} + \overline{\alpha}_1 H \geq c_T \mathbf{1}'\mathbf{K}_\mathbf{T}^\mathbf{1} + F_T$,

iii) increases in credit limit ($\frac{\partial}{\partial E}\mathbf{E}[\pi_T(E, B_{FRM}(\alpha_1, H))] \geq 0$), *and the equality only holds for H such that* $\omega_0^{FRM} + \overline{\alpha}_1 H \geq c_T \mathbf{1}'\mathbf{K}_\mathbf{T}^\mathbf{1} + F_T - E$,

iv) increases in demand variability ($\frac{\partial}{\partial \sigma}\mathbf{E}[\pi_T(E, B_{FRM}(\alpha_1, H))] \geq 0$),

v) decreases in demand correlation ($\frac{\partial}{\partial \rho}\mathbf{E}[\pi_T(E, B_{FRM}(\alpha_1, H))] \leq 0$).

Since the expected (stage 0) equity value is a continuous function of parameters $a, E, F_T, \gamma_T, \rho, \sigma$ for a given financial risk management level H, the expected (stage 0) equity value at the optimal risk management level (which also depends on these parameters) is also continuous in these parameters. Therefore the monotonic relations stated in Lemma 6 are also satisfied in the weak sense (not strict inequality) at the optimal financial risk management level without assuming differentiability (because the expected (stage 0) equity value might not be differentiable at the points where the optimal financial risk management level changes). The comparative static results for $\overline{c}_F(c_D, \mathbf{H}^*)$ follow from Lemma 6. The comparative static results for $\overline{c}_F(c_D, \mathbf{0})$ also follow from Lemma 6 using the identity $B_{-FRM}(\alpha_1) = B_{FRM}(\alpha_1, H)$ for $H = 0$ and $F_{FRM} = 0$.

With symmetric fixed costs and salvage rates, we establish the functional form of $\overline{c}_F^S(c_D)$ with the following Lemma and relegate the proof to Appendix 5:

Lemma 7 *When the fixed costs and the salvage rates of the two technologies are symmetric, at* $c_F = \overline{c}_F^S(c_D)$ *expected (stage 1) equity values, expected (stage 0) equity values at an arbitrary financial risk management level H and the optimal financial risk management actions are the same for both technologies, i.e.* $\pi_F(c_F, \tilde{B})\Big|_{c_F = \overline{c}_F^S(c_D)} = \pi_D(c_D, \tilde{B})$ *for* $\tilde{B} \geq 0$, $\mathbf{E}[\pi_F(\overline{c}_F^S(c_D), B_{FRM}(\alpha_1, H))] = \mathbf{E}[\pi_D(c_D, B_{FRM}(\alpha_1, H))]$ *and* $H_F^*(\overline{c}_F^S(c_D)) = H_D^*(c_D)$.

It follows from Lemma 7 that $\overline{c}_F^S(c_D)$ is the unique threshold with financial risk management in the symmetric case ($\overline{c}_F(c_D, \mathbf{H}^*) = \overline{c}_F^S(c_D)$). Using the identity $B_{-FRM}(\alpha_1) =$

$B_{FRM}(\alpha_1, H)$ for $H = 0$ and $F_{FRM} = 0$, it follows from Lemma 7 that $\bar{c}_F^S(c_D)$ is also the unique threshold without financial risk management in the symmetric case ($\bar{c}_F(c_D, \mathbf{0}) = \bar{c}_F^S(c_D)$). We now prove the relation $\bar{c}_F^S(c_D) \geq c_D$. It is sufficient to show

$$\mathbf{E}^{-b}\left[\left(\xi_1^{-b} + \xi_2^{-b}\right)^{-\frac{1}{b}}\right] \geq \mathbf{E}^{-b}[\xi_1] + \mathbf{E}^{-b}[\xi_2].$$

From Hardy et al. (1988, p.133,146) if $d \in (0,1)$ and X and Y are non-negative random variables then the following is true:

$$E^{1/d}\left[(X+Y)^d\right] \geq \mathbf{E}^{1/d}[X^d] + \mathbf{E}^{1/d}[Y^d] \tag{5.35}$$

where the equality only holds when X and Y are effectively proportional, i.e. $X = \lambda Y$. In the expression for $\bar{c}_F^S(c_D)$ we have $d = -\frac{1}{b} \in (0,1)$ and $\boldsymbol{\xi} > \mathbf{0}$ therefore we can use this inequality. Replacing X with ξ_1^{-b} and Y with ξ_2^{-b} gives the desired result. Notice that $\bar{c}_F^S(c_D) = c_D$ only if $\xi_1 = k\xi_2$ for $k > 0$. This is only possible if either $\boldsymbol{\xi}$ is deterministic or it is perfectly positively correlated and has a proportional bivariate distribution. ■

Proof of Corollary 2: If the capital markets are perfect we have $E = P \geq c_T \mathbf{1}'\mathbf{K}_\mathbf{T}^\mathbf{0} + F_T$ and $a = 0$ (as we discussed in Assumption 15). Since we have $\Omega_T^{1234} = \emptyset$, it follows from Proposition 2 that the firm invests in the budget-unconstrained capacity investment level for any budget realization, $\mathbf{K}_\mathbf{T}^*(\tilde{B}) = \mathbf{K}_\mathbf{T}^\mathbf{0}$, and borrows to finance this capacity level, $e_T^*(\tilde{B}) = [c_T\mathbf{1}'\mathbf{K}_\mathbf{T}^\mathbf{0} + F_T - B]^+$. We obtain

$$\mathbf{E}\left[\pi_T(B_{-FRM}(\alpha_1))\right] = \mathbf{E}\left[\pi_T(B_{FRM}(\alpha_1, H))\right]|_{F_{FRM}=0} = \omega_0 + \overline{\alpha}_1\omega_1 - (1-\gamma_T)F_T + \frac{c_T\mathbf{1}'\mathbf{K}_\mathbf{T}^\mathbf{0}}{-(b+1)} + P,$$

and it follows from Proposition 30 that $\overline{F}_{FRM}^T = 0$ for $T \in \{D, F\}$. If the product markets are perfect ($\mathbf{\Sigma} = \mathbf{0}$), then with symmetric fixed costs and salvage rates, it follows from Proposition 4 that $\bar{c}_F(c_D, \mathbf{H}^*) = \bar{c}_F(c_D, \mathbf{0}) = c_D$. ■

Proof of Corollary 3: The proof of the first argument follows from Proposition 3. For the second argument, we provide a numerical example where the firm optimally fully speculates with flexible technology and fully hedges with dedicated technology. We focus on the case with $F_{FRM} = 0$ such that financial risk management is costless. The horizontal line in Figure 5.1 denotes the value of not investing any technology; hence the firm optimally chooses flexible technology with full speculation in this example. ■

Proof of Proposition 5: With a hedging constraint, the range of forward contracts is $[0, \omega_1^{FRM}]$ in (5.30). Substituting $F_{FRM} = 0$ in (5.33) of Proposition 3, similar to (5.34), we obtain $\frac{\partial \pi_T}{\partial H_T} < 0$. It follows that $H_T^* = 0$. ■

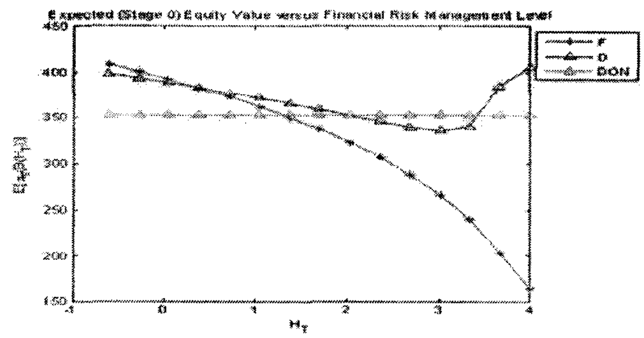

Figure 5.1: Optimal Speculation is triggered by flexible technology investment: Dedicated technology with full hedging ($H_D^* = \omega_1 = 4$) is dominated by flexible technology with full speculation ($H_F^* = -\frac{\omega_0}{\alpha_1} = -0.61$.

Proof of Corollary 4: It follows from Proposition 4 that for symmetric fixed costs and salvage rates of technologies and for $F_{FRM} = 0$, the optimal risk management portfolio is flexible (dedicated) technology with financial risk management if $c_F < \bar{c}_F^S(c_D)$ ($c_F > \bar{c}_F^S(c_D)$). From the proof of Proposition 30, for $\beta = \frac{\omega_0}{\omega_0 + \alpha_0 \omega_1}$ we can have a sufficiently large feasible F_{FRM} such that engaging in financial risk management is not profitable. In this case, the optimal risk management portfolio is flexible (dedicated) technology with financial risk management if $c_F < \bar{c}_F^S(c_D)$ ($c_F > \bar{c}_F^S(c_D)$). ■

Proof of Proposition 6: The invariance of $\bar{c}_F(c_D, \mathbf{H}^*)$ and $\bar{c}_F(c_D, \mathbf{0})$ to the unit financing cost, the fixed cost of both technologies and the internal endowment of the firm follows from the definition of $c_F^S(c_D)$ in Proposition 4. For $F = F_D < F_F = F + \delta$ with $\delta > 0$, we obtain $\bar{c}_F(c_D, \mathbf{H}^*) < \bar{c}_F^S(c_D)$ and $\bar{c}_F(c_D, \mathbf{0}) < \bar{c}_F^S(c_D)$ from Proposition 4. We first provide the proof of the results with respect to technology fixed costs. Comparative statics with respect to the internal endowment follow from a similar argument. We define

$$S^{-FRM}(c_F) \doteq \mathbf{E}[\pi_F(c_F, F + \delta, B_{-FRM}(\alpha_1))] - \mathbf{E}[\pi_D(c_D, F, B_{-FRM}(\alpha_1))] \text{ where } S^{-FRM}(\bar{c}_F(c_D, \mathbf{0})) = 0 \quad (5.36)$$

From the implicit function theorem we have $\frac{\partial}{\partial F}\bar{c}_F(c_D, \mathbf{0}) = -\frac{\partial}{\partial F}S^{-FRM}\left(\frac{\partial}{\partial c_F}S^{-FRM}\right)^{-1}\Big|_{\bar{c}_F(c_D, \mathbf{0})}$.
From Lemma 2, we can interchange derivative and expectation operators, and using Lemma 3 with $B_{-FRM}(\alpha_1) = B_{FRM}(\alpha_1, H)$ for $H = 0$ and $F_{FRM} = 0$, we obtain

$$\frac{\partial S^{-FRM}}{\partial c_F}\Big|_{\bar{c}_F(c_D, \mathbf{0})} = \mathbf{E}\left[\frac{\partial \pi_F(B_{-FRM}(\alpha_1))}{\partial c_F}\right]\Big|_{\bar{c}_F(c_D, \mathbf{0})} < 0.$$

Similarly we have

$$\frac{\partial S^{-FRM}}{\partial F}\Big|_{\bar{c}_F(c_D,\mathbf{0})} = \left[\mathbf{E}\left[\frac{\partial \pi_F(B_{-FRM}(\alpha_1))}{\partial F}\right] - \mathbf{E}\left[\frac{\partial \pi_D(B_{-FRM}(\alpha_1)))}{\partial F}\right]\right]\Big|_{\bar{c}_F(c_D,\mathbf{0})}.$$

Since $\bar{c}_F(c_D,\mathbf{0}) < \bar{c}_F^S$ and $\delta > 0$ it follows that $c_F\mathbf{K_F^i}|_{\bar{c}_F(c_D,\mathbf{0})} + F + \delta > c_D\mathbf{1}'\mathbf{K_D^i} + F$ for $i = 0, 1$. This implies that $\Omega_F^0 \subset \Omega_D^0$ and $\Omega_F^2 \supset \Omega_D^2$. We obtain

$$\begin{aligned}
\frac{\partial S^{-FRM}}{\partial F}\Big|_{\bar{c}_F(c_D,\mathbf{0})} &= \int_{\Omega_F^0 \bigcap \Omega_D^0} (-1+1) \ \ dR_{B_{-FRM}}(\tilde{B}) \qquad (5.37\\
&+ \int_{\Omega_F^1 \bigcap \Omega_D^0} \left(-\frac{M_F\left(1+\frac{1}{b}\right)}{\bar{c}_F(c_D,\mathbf{0})}\left(\frac{\tilde{B}-F-\delta}{\bar{c}_F(c_D,\mathbf{0})}\right)^{\frac{1}{b}} + 1\right) \ dR_{B_{-FRM}}(\tilde{B})\\
&+ \int_{\Omega_F^1 \bigcap \Omega_D^1} \left(1+\frac{1}{b}\right)\left[-\frac{M_F}{(\bar{c}_F(c_D,\mathbf{0}))^{1+\frac{1}{b}}}\left(\tilde{B}-F-\delta\right)^{\frac{1}{b}} + \frac{M_D}{c_D^{1+\frac{1}{b}}}\left(\tilde{B}-F\right)^{\frac{1}{b}}\right] dR_{B_{-FRM}}(\tilde{B}\\
&+ \int_{\Omega_F^2 \bigcap \Omega_D^0} (-(1+a)+1) \ \ dR_{B_{-FRM}}(\tilde{B})\\
&+ \int_{\Omega_F^2 \bigcap \Omega_D^1} \left[-(1+a) + (1+\frac{1}{b})\frac{M_D}{c_D^{1+\frac{1}{b}}}\left(\tilde{B}-F\right)^{\frac{1}{b}}\right] \ dR_{B_{-FRM}}(\tilde{B})\\
&+ \int_{\Omega_F^2 \bigcap \Omega_D^2} (-(1+a)+(1+a)) \ \ dR_{B_{-FRM}}(\tilde{B}).
\end{aligned}$$

From (5.26), we have $\frac{\partial}{\partial F}S^{-FRM}|_{\bar{c}_F(c_D,\mathbf{0})} < 0$ for $\tilde{B} \in \Omega_F^1 \bigcap \Omega_D^0$ and $\tilde{B} \in \Omega_F^2 \bigcap \Omega_D^1$. From Lemma 7, we have $\frac{M_D}{c_D^{1+\frac{1}{b}}} = \frac{M_F}{(\bar{c}_F^S(c_D))^{1+\frac{1}{b}}}$. Since $\bar{c}_F(c_D,\mathbf{0}) < \bar{c}_F^S(c_D)$ and $\delta > 0$, we obtain $\frac{\partial}{\partial F}S^{-FRM}|_{\bar{c}_F(c_D,\mathbf{0})} < 0$ for $\tilde{B} \in \Omega_F^1 \bigcap \Omega_D^1$. In conclusion, we have $\frac{\partial}{\partial c_F}S^{-FRM}\Big|_{\bar{c}_F(c_D,\mathbf{0})} < 0$ and $\frac{\partial}{\partial F}S^{-FRM}|_{\bar{c}_F(c_D,\mathbf{0})} \leq 0$. It follows from the implicit function theorem that $\frac{\partial}{\partial F}\bar{c}_F(c_D,\mathbf{0}) \leq 0$ where the equality holds only for $\omega_0 > c_F\mathbf{K_F^0}|_{\bar{c}_F(c_D,\mathbf{0})} + F + \delta$.

To prove the result for $\bar{c}_F(c_D,\mathbf{H}^*)$, we define $S^{FRM}(c_F)$, the counterpart of (5.36) by replacing $B_{-FRM}(\alpha_1)$ with $B_{FRM}(\alpha_1, H_T^*)$. We have $H_F^*(c_F) = H_D^* = \omega_1^{FRM}$ for $c_F = \bar{c}_F(c_D,\mathbf{H}^*)$. We establish $\frac{\partial}{\partial c_F}S^{FRM}\Big|_{\bar{c}_F(c_D,\mathbf{H}^*)} < 0$ using $\frac{\partial}{\partial c_F}H_F^*\Big|_{\bar{c}_F(c_D,\mathbf{H}^*)} = 0$. The rest of the proof follows in a similar manner using the facts that $\frac{\partial}{\partial F}H_T^*|_{\bar{c}_F(c_D,\mathbf{H}^*)} = 0$ and that with full-hedging $\omega_0^{FRM} + \bar{\alpha}_1\omega_1^{FRM}$ is realized in only one of the regions in (5.37). In conclusion, it follows from the implicit function theorem that $\frac{\partial}{\partial F}\bar{c}_F(c_D,\mathbf{H}^*) \leq 0$ where the equality holds only for $\omega_0^{FRM} + \bar{\alpha}_1\omega_1^{FRM} \in \Omega_F^0 \bigcap \Omega_D^0$ or $\omega_0^{FRM} + \bar{\alpha}_1\omega_1^{FRM} \in \Omega_F^2 \bigcap \Omega_D^2$.

To prove the results with respect to the unit financing cost for $\bar{c}_F(c_D,\mathbf{0})$, we follow the same steps by replacing F with a in $S^{-FRM}(c_F)$. We obtain

$$\begin{aligned}\left.\frac{\partial S^{-FRM}}{\partial a}\right|_{\bar{c}_F(c_D,0)} &= \int_{\Omega_F^2\setminus\Omega_D^2}\left(\tilde{B}-(c_F\mathbf{K_F^{\frac{1}{b}}}+F_F)|_{\bar{c}_F(c_D,0)}\right)\; dR_{B_{-FRM}}(\tilde{B}) \\ &+ \int_{\Omega_F^2\bigcap\Omega_D^2}\left(c_D\mathbf{1'K_D^{\frac{1}{b}}}+F_D-(c_F\mathbf{K_F^{\frac{1}{b}}}+F_F)|_{\bar{c}_F(c_D,0)}\right)\; dR_{B_{-FRM}}(\tilde{B}) \\ &+ \int_{\Omega_F^3\setminus\Omega_D^{23}} -E\; dR_{B_{-FRM}}(\tilde{B}) \\ &+ \int_{\Omega_F^3\bigcap\Omega_D^2}\left(-\tilde{B}+(c_F\mathbf{K_F^{\frac{1}{b}}}+F_F)|_{\bar{c}_F(c_D,0)}-E\right)\; dR_{B_{-FRM}}(\tilde{B}).\end{aligned} \tag{5.38}$$

The first term and the last integrands are negative by the definition of the regions. From above (comparative static with respect to fixed cost) we have $c_D\mathbf{1'K_D^{\frac{1}{b}}}+F_D < c_F\mathbf{K_F^{\frac{1}{b}}}|_{\bar{c}_F(c_D,0)}+F_F$. This implies $\frac{\partial}{\partial a}S^{-FRM}|_{\bar{c}_F(c_D,0)} \leq 0$. We conclude $\frac{\partial}{\partial a}\bar{c}_F(c_D,\mathbf{0}) \geq 0$ where the equality holds for $\omega_0 > c_F\mathbf{K_F^{\frac{1}{b}}}|_{\bar{c}_F(c_D,0)}+F_F$.

The result for $\bar{c}_F(c_D,\mathbf{H}^*)$ can be proven in a similar fashion. It follows that $\frac{\partial}{\partial a}\bar{c}_F(c_D,\mathbf{H}^*) \geq 0$ where the equality holds if $\omega_0^{FRM}+\overline{\alpha}_1\omega_1^{FRM} \in \Omega_F^{01}\bigcup(\Omega_F^3\bigcap\Omega_D^3)$. ∎

Proof of Proposition 7: We only prove the results for small firms. Results related to large firms follow from similar arguments. We define

$$\Upsilon^{\varphi} \doteq \frac{\partial\Delta_T}{\partial\varphi} = \frac{\partial\mathbf{E}\left[\pi_T\left(B_{FRM}(\alpha_1,\omega_1^{FRM})\right)\right]}{\partial\varphi} - \frac{\partial\mathbf{E}\left[\pi_T(B_{-FRM}(\alpha_1))\right]}{\partial\varphi} \tag{5.39}$$

as the derivative of the value of full hedging with respect to the argument φ. For small firms, we have $\mathbf{E}\left[\pi_T\left(B_{FRM}(\alpha_1,\omega_1^{FRM})\right)\right] = (\omega_0^{FRM}+\overline{\alpha}_1\omega_1^{FRM}-F_T)(1+a)+\frac{c_T\mathbf{1'K_T^{\frac{1}{b}}}(1+a)}{-(b+1)}+\gamma_T F_T+P$. We analyze each comparative static result separately.

Fixed cost of technology. We obtain

$$\begin{aligned}\Upsilon^{F_T} = -(1+a) - \int_{\Omega_T^0} -1\; dR_{B_{-FRM}}(\tilde{B}) \;&-\; \int_{\Omega_T^1}(1+\frac{1}{b})\frac{M_T}{c_T}\left(\frac{\tilde{B}-F_T}{c_T}\right)^{\frac{1}{b}} dR_{B_{-FRM}}(\tilde{B}) \\ &-\; \int_{\Omega_T^2} -(1+a)\; dR_{B_{-FRM}}(\tilde{B}).\end{aligned}$$

It is easy to show that $(1+\frac{1}{b})\frac{M_T}{c_T}\left(\frac{\tilde{B}-F_T}{c_T}\right)^{\frac{1}{b}} < 1+a$ for $\tilde{B}\in\Omega_T^1$. It follows that $\Upsilon^{F_T}<0$.

Initial endowment. After parameterizing the initial endowment, we obtain $\beta^{\lambda} = \frac{\lambda\omega_0}{\lambda\omega_0+\alpha_0\lambda\omega_1} = \frac{\omega_0}{\omega_0+\alpha_0\omega_1} = \beta$. We have $(\omega_0^{FRM},\omega_1^{FRM}) \doteq (\lambda\omega_0-\beta F_{FRM},\lambda\omega_1-\frac{1-\beta}{\alpha_0}F_{FRM})$ and for small firms, it follows that $\frac{\partial\mathbf{E}\left[\pi_T\left(B_{FRM}(\alpha_1,\omega_1^{FRM})\right)\right]}{\partial\lambda} = (\omega_0+\overline{\alpha}_1\omega_1)(1+a)$. After parameterizing the initial endowment, we define $\alpha_T^0 \doteq \frac{\frac{c_T \mathbf{1}'\mathbf{K}_\mathbf{T}^0 + F_T}{\lambda} - \omega_0}{\omega_1}$, $\alpha_T^1 \doteq \frac{\frac{c_T \mathbf{1}'\mathbf{K}_\mathbf{T}^1 + F_T}{\lambda} - \omega_0}{\omega_1}$. We obtain

$$\begin{aligned}
\Upsilon^\lambda &= (\omega_0 + \overline{\alpha}_1\omega_1)(1+a) - \int_{\max(\alpha_T^0,0)}^{\infty} (\omega_0 + x\omega_1)\, r_{\alpha_1}(x)\, dx \\
&- \int_{\max(\alpha_T^1,0)}^{\max(\alpha_T^0,0)} \frac{M_T(1+1/b)}{c_T} \left(\frac{\lambda(\omega_0 + x\omega_1) - F_T}{c_T}\right)^{\frac{1}{b}} (\omega_0 + x\omega_1)\, r_{\alpha_1}(x)\, dx \\
&- \int_0^{\max(\alpha_T^1,0)} (\omega_0 + x\omega_1)(1+a)\, r_{\alpha_1}(x)\, dx
\end{aligned}$$

Notice that negative terms above are the expected value of the following function

$$f(\alpha_1) = \begin{cases} \omega_0 + \alpha_1\omega_1 & \text{if } \alpha_1 \geq \alpha_T^0 \\ \frac{M_T(1+1/b)}{c_T}\left(\frac{\lambda(\omega_0+\alpha_1\omega_1)-F_T}{c_T}\right)^{\frac{1}{b}}(\omega_0 + \alpha_1\omega_1) & \text{if } \alpha_T^0 > \alpha_1 \geq \alpha_T^1 \\ (\omega_0 + \alpha_1\omega_1)(1+a) & \text{if } \alpha_1 < \alpha_T^1 \end{cases}$$

with respect to the asset price distribution α_1. It is easy to prove that $(\omega_0+\alpha_1\omega_1)(1+a) \geq f(\alpha_1)$ for $\alpha_1 \geq 0$ with strict inequality for some α_1. It follows that $\mathbf{E}[(\omega_0+\alpha_1\omega_1)(1+a)] = (\omega_0+\overline{\alpha}_1\omega_1)(1+a) > \mathbf{E}[f(\alpha_1)]$ and we obtain $\Upsilon^\lambda > 0$.

To analyze the effect of cash holdings (ω_0) on the value of financial risk management, we only parameterize the cash holdings as $(\lambda'\omega_0, \omega_1)$ and set $\beta = 0$ such that F_{FRM} is only deducted from the value of asset holdings ω_1. It follows that $\omega_0^{FRM} = \lambda'\omega_0$ and $\omega_1^{FRM} = \omega_1 - \frac{F_{FRM}}{\alpha_0}$. $\Upsilon^{\lambda'} > 0$ follows from the similar lines with $\Upsilon^\lambda > 0$.

Demand variability and correlation. We only provide the proof for demand variability. The proof for demand correlation is along the similar lines. It is sufficient to focus on flexible technology because dedicated technology is not affected from changes in σ and ρ. We obtain

$$\begin{aligned}
\Upsilon^\sigma &= \frac{\partial M_F}{\partial \sigma}\left(\frac{c_F M_F(1+\frac{1}{b})}{1+a}\right)^{-b-1} - \int_{\Omega_F^0} \frac{\partial M_F}{\partial \sigma}\left(c_F M_F(1+\frac{1}{b})\right)^{-b-1} dR_{B_{-FRM}}(\tilde{B}) \\
&- \int_{\Omega_F^1} \frac{\partial M_F}{\partial \sigma}\left(\frac{\tilde{B}-F_F}{c_F}\right)^{1+\frac{1}{b}} dR_{B_{-FRM}}(\tilde{B}) - \int_{\Omega_F^2} \frac{\partial M_F}{\partial \sigma}\left(\frac{c_F M_F(1+\frac{1}{b})}{1+a}\right)^{-b-1} dR_{B_{-FRM}}(\tilde{B}).
\end{aligned}$$

It is easy to show $\left(\frac{\tilde{B}-F_F}{c_F}\right)^{1+\frac{1}{b}} > \left(\frac{c_F M_F(1+\frac{1}{b})}{1+a}\right)^{-b-1}$ for $\tilde{B} \in \Omega_F^1$. From Lemma 4, we have $\frac{\partial}{\partial\sigma}M_F \geq 0$ and it follows that $\Upsilon^\sigma \leq 0$.

Unit financing cost. We obtain

$$\Upsilon^a = \omega_0 + \overline{\alpha}_1\omega_1 - (\beta + \frac{(1-\beta)\overline{\alpha}_1}{\alpha_0})F_{FRM} - c_T\mathbf{1}'\mathbf{K}_\mathbf{T}^1 - F_T - \int_{\Omega_T^2} (\tilde{B} - c_T\mathbf{1}'\mathbf{K}_\mathbf{T}^1 - F_T)\ dR_{B_{-FRM}}(\tilde{B}).$$

It follows that for $\omega_0 > c_T\mathbf{1}'\mathbf{K}_\mathbf{T}^1 + F_T$, when the non-hedged firm does not borrow at all, we have $\Upsilon^a < 0$. We focus on the case where the firm borrows at some budget states without financial risk management ($\omega_0 < c_T\mathbf{1}'\mathbf{K}^{\mathbf{1}}_{\mathbf{T}} + F_T$).

For $F_{FRM} = 0$, we have $\Upsilon^a = \int_{\Omega^2_T}(c_T\mathbf{1}'\mathbf{K}^{\mathbf{1}}_{\mathbf{T}} + F_T - \tilde{B})\ dR_{B_{-FRM}}(\tilde{B}) - (c_T\mathbf{1}'\mathbf{K}^{\mathbf{1}}_{\mathbf{T}} + F_T - \overline{B})$ where $\overline{B} = \omega_0 + \overline{\alpha}_1\omega_1$. Notice that the first term is the expected value of the function

$$f(\tilde{B}) = \begin{cases} c_T\mathbf{1}'\mathbf{K}^{\mathbf{1}}_{\mathbf{T}} + F_T - \overline{B} & \text{if } \tilde{B} \le c_T\mathbf{1}'\mathbf{K}^{\mathbf{1}}_{\mathbf{T}} + F_T \\ 0 & \text{if } \tilde{B} > c_T\mathbf{1}'\mathbf{K}^{\mathbf{1}}_{\mathbf{T}} + F_T \end{cases}$$

with respect to the budget distribution. Since $f(\tilde{B})$ is a convex function, $\Upsilon^a \ge 0$ for $F_{FRM} = 0$ follows from Jensen's inequality.

For $F_{FRM} > 0$, we have

$$\Upsilon^a = \int_{\Omega^2_T}(c_T\mathbf{1}'\mathbf{K}^{\mathbf{1}}_{\mathbf{T}} + F_T - \tilde{B})\ dR_{B_{-FRM}}(\tilde{B}) - (c_T\mathbf{1}'\mathbf{K}^{\mathbf{1}}_{\mathbf{T}} + F_T + (\beta + \frac{(1-\beta)\overline{\alpha}_1}{\alpha_0})F_{FRM} - \overline{B}).$$

We observe that the first term is strictly less than $c_T\mathbf{1}'\mathbf{K}^{\mathbf{1}}_{\mathbf{T}} + F_T - \omega_0$. For $F_{FRM} \ge F^0_{FRM} = \frac{\alpha_0\omega_1}{(1-\beta)+\frac{\beta}{\overline{\alpha}_1}}$, we obtain $c_T\mathbf{1}'\mathbf{K}^{\mathbf{1}}_{\mathbf{T}} + F_T + (\beta + \frac{(1-\beta)\overline{\alpha}_1}{\alpha_0})F_{FRM} - \overline{B} \ge c_T\mathbf{1}'\mathbf{K}^{\mathbf{1}}_{\mathbf{T}} + F_T - \omega_0$ and it follows that $\Upsilon^a < 0$. Notice that $F_{FRM} \le \frac{\alpha_0\omega_1}{(1-\beta)}$ is the feasiblity condition; hence such F_{FRM} exists. We calculate $\frac{\partial}{\partial F_{FRM}}\Upsilon^a = -(\beta + \frac{(1-\beta)\overline{\alpha}_1}{\alpha_0}) < 0$. Since Υ^a strictly decreases in F_{FRM}, $\Upsilon^a \ge 0$ for $F_{FRM} = 0$ and $\Upsilon^a < 0$ for F^0_{FRM}, we conclude that there exists a unique $\widehat{F}_{FRM}$ such that $\Upsilon^a < 0$ for $F_{FRM} > \widehat{F}_{FRM}$ and $\Upsilon^a \ge 0$ for $F_{FRM} \le \widehat{F}_{FRM}$. ■

Proof of Proposition 8: We focus on the case where it is profitable for the firm to engage in financial risk management. To prove the proposition, we use the ordering between $\overline{c}_F(c_D, \mathbf{H}^*)$ and $\overline{c}_F(c_D, 0)$. If $\overline{c}_F(c_D, \mathbf{0}) < \overline{c}_F(c_D, \mathbf{H}^*)$ ($\overline{c}_F(c_D, \mathbf{0}) > \overline{c}_F(c_D, \mathbf{H}^*)$) then flexible technology and financial risk management are complements (substitutes) because engaging in financial risk management enables the firm to invest in flexible (dedicated) technology at some technology cost levels where dedicated (flexible) technology was more profitable without financial risk management. From Proposition 4, we obtain $\overline{c}_F(c_D, \mathbf{H}^*) < \overline{c}^S_F(c_D)$ and $\overline{c}_F(c_D, \mathbf{0}) < \overline{c}^S_F(c_D)$. From Assumption 8, we have $H^*_D(c_D) = H^*_F(\overline{c}_F(c_D, \mathbf{H}^*)) = \omega^{FRM}_1$. From Lemma 3, it follows that $\overline{c}_F(c_D, \mathbf{H}^*) \gtrless \overline{c}_F(c_D, \mathbf{0})$ if and only if

$$\mathbf{E}\left[\pi_D(B_{FRM}(\alpha_1, \omega^{FRM}_1))\right] \lesseqgtr \mathbf{E}\left[\pi_F(\overline{c}_F(c_D, \mathbf{0}), B_{FRM}(\alpha_1, \omega^{FRM}_1)))\right]. \tag{5.40}$$

Recall that $\triangle_T(c_T, F_T)$ is the value of financial risk management with technology $T \in \{D, F\}$ at given cost parameters (c_T, F_T) as defined in (2.5). Inequality (5.40) holds if and only if $\triangle_F(\overline{c}_F(c_D, \mathbf{0}), F_F) \gtrless \triangle_D(c_D, F_D)$. We will use the relation between $\triangle_F(\overline{c}_F(c_D, \mathbf{0}), F_F)$ and $\triangle_D(c_D, F_D)$ to prove the proposition. We provide the following lemma and relegate the proof to Appendix 5.

Lemma 8 *For $F_T < \overline{F}_T$ and $E > c_T \mathbf{1}'\mathbf{K}_T^1 + F_T$,*

(i) If $\omega_0^{FRM} + \overline{\alpha}_1\omega_1^{FRM} \in \Omega_T^0$ ($\in \Omega_T^2$) then $\frac{\partial}{\partial F_T}\triangle_T \geq 0$ ($\frac{\partial}{\partial F_T}\triangle_T < 0$);

(ii) If $\omega_0^{FRM} + \overline{\alpha}_1\omega_1^{FRM} \in \Omega_T^0$ ($\in \Omega_T^2$) then $\frac{\partial}{\partial c_T}\triangle_T \leq 0$ ($\frac{\partial}{\partial c_T}\triangle_T > 0$).

For large firms ($\omega_0^{FRM} + \overline{\alpha}_1\omega_1^{FRM} \in \Omega_F^0$), we obtain from Lemma 8, $\overline{c}_F^S(c_D) > \overline{c}_F(c_D, \mathbf{0})$ and $F_F \geq F_D$ that

$$\triangle_D(c_D, F_D) = \triangle_F(\overline{c}_F^S(c_D), F_D) \leq \triangle_F(\overline{c}_F(c_D, \mathbf{0}), F_D) \leq \triangle_F(\overline{c}_F(c_D, \mathbf{0}), F_F).$$

From the proof of Lemma 8, the inequalities above are strict for sufficiently low ω_0. We conclude that $\overline{c}_F(c_D, \mathbf{H}^*) \geq \overline{c}_F(c_D, \mathbf{0})$ and large firms tend to use flexible technology and financial risk management as complements.

For small firms ($\omega_0^{FRM} + \overline{\alpha}_1\omega_1^{FRM} \in \Omega_F^2$), we obtain

$$\triangle_D(c_D, F_D) = \triangle_F(\overline{c}_F^S(c_D), F_D) > \triangle_F(\overline{c}_F(c_D, \mathbf{0}), F_D) > \triangle_F(\overline{c}_F(c_D, \mathbf{0}), F_F).$$

We conclude that $\overline{c}_F(c_D, \mathbf{H}^*) < \overline{c}_F(c_D, \mathbf{0})$ and small firms tend to substitute flexible technology with financial risk management. ∎

Proof of Proposition 9: We only prove the results for small firms. Results related to large firms follow from similar arguments. Recall that in the proof of Proposition 6 we defined

$$\begin{aligned} S^{FRM} &= \mathbf{E}\left[\pi_F\left(B_{FRM}(\alpha_1, \omega_1^{FRM})\right)\right] - \mathbf{E}\left[\pi_D\left(B_{FRM}(\alpha_1, \omega_1^{FRM})\right)\right] \\ S^{-FRM} &= \mathbf{E}\left[\pi_F\left(B_{-FRM}(\alpha_1)\right)\right] - \mathbf{E}\left[\pi_D\left(B_{-FRM}(\alpha_1)\right)\right] \end{aligned}$$

as the value of operational risk management with and without financial risk management respectively. The value of operational risk management is more robust to a change in $\varphi \in \{a, \rho, \sigma\}$ with financial risk management then without if

$$\left|\frac{\partial\, S^{FRM}}{\partial \varphi}\right| < \left|\frac{\partial\, S^{-FRM}}{\partial \varphi}\right|.$$

To analyze the robustness of the value of operational risk management, we focus on the cases where operational risk management has a value, i.e. flexible technology is preferred over dedicated technology with and without financial risk management. Recall from the proof of Proposition 6 that we have $\overline{c}_F(c_D, \mathbf{H}^*) < \overline{c}_F^S(c_D)$ and $\overline{c}_F(c_D, \mathbf{0}) < \overline{c}_F^S(c_D)$ in this setting. Therefore, for any relevant unit investment cost pair (c_F, c_D) we have $c_F < \overline{c}_F^S(c_D)$. We now analyze each market condition separately.

Robustness with respect to capital market condition (a). Since $c_F < \bar{c}_F^S(c_D)$, it follows from (5.38) that $\frac{\partial}{\partial a}S^{FRM} \leq 0$ and $\frac{\partial}{\partial a}S^{-FRM} \leq 0$. Therefore, it is sufficient to show $\frac{\partial}{\partial a}S^{FRM} \leq \frac{\partial}{\partial a}S^{FRM}$ to prove the result of lower robustness. It follows from (5.39) that this condition is equivalent to $\frac{\partial}{\partial a}\Delta_F \leq \frac{\partial}{\partial a}\Delta_D$. We obtain

$$\frac{\partial \Delta_F}{\partial a} - \frac{\partial \Delta_D}{\partial a} = \omega_0 + \overline{\alpha}_1\omega_1 - s(\overline{\alpha}_1)F_{FRM} - c_F\mathbf{K_F^1} - F_F - \int_{\Omega_F^2} (\tilde{B} - c_F\mathbf{K_F^1} - F_F)\ dR_{B\_FRM}(\tilde{B})$$
$$- \quad [\omega_0 + \overline{\alpha}_1\omega_1 - s(\overline{\alpha}_1)F_{FRM} - c_D\mathbf{1'K_D^1} - F_D]\chi(\overline{B} \in \Omega_D^2) + \int_{\Omega_D^2} (\tilde{B} - c_D\mathbf{1'K_D^1} - F_D)\ dR_{B\_FRM}(\tilde{B}).$$

where $s(\overline{\alpha}_1) = \beta + \frac{(1-\beta)\overline{\alpha}_1}{\alpha_0}$, $\overline{B} = \omega_0 + \overline{\alpha}_1\omega_1 - s(\overline{\alpha}_1)F_{FRM}$ and $\chi(.)$ is the indicator function. We have the indicator function because a small firm (that always borrows with financial risk management with flexible technology) need not to borrow with financial risk management with dedicated technology. We now show that $\frac{\partial}{\partial a}\Delta_F \leq \frac{\partial}{\partial a}\Delta_D$ by focusing on two cases.
Case i : $(\overline{B} \in \Omega_D^2)$ We obtain

$$\begin{aligned}\frac{\partial \Delta_F}{\partial a} - \frac{\partial \Delta_D}{\partial a} \quad &= \int_{\Omega_F^2\backslash\Omega_D^2} (c_F\mathbf{K_F^1} + F_F - \tilde{B})\ dR_{B\_FRM}(\tilde{B}) \\ &+ \int_{\Omega_F^2 \bigcap \Omega_D^2} (c_F\mathbf{K_F^1} + F_F - c_D\mathbf{1'K_D^1} - F_D)\ dR_{B\_FRM}(\tilde{B}) - (c_F\mathbf{K_F^1} + F_F - c_D\mathbf{1'K_D^1} - F_D).\end{aligned}$$

Since for $\tilde{B} \in \Omega_F^2\backslash\Omega_D^2$ we have $\tilde{B} \geq c_D\mathbf{1'K_D^1} + F_D$, it follows that $\frac{\partial}{\partial a}\Delta_F < \frac{\partial}{\partial a}\Delta_D$.
Case ii : $(\overline{B} \in \Omega_D^{01})$ We obtain

$$\frac{\partial \Delta_F}{\partial a} - \frac{\partial \Delta_D}{\partial a} = \int_{\Omega_F^2\backslash\Omega_D^2} (c_F\mathbf{K_F^1} + F_F - \tilde{B})\ dR_{B\_FRM}(\tilde{B})$$
$$+ \int_{\Omega_F^2 \bigcap \Omega_D^2} (c_F\mathbf{K_F^1} + F_F - c_D\mathbf{1'K_D^1} - F_D)\ dR_{B\_FRM}(\tilde{B}) - (c_F\mathbf{K_F^1} + F_F - \omega_0 - \overline{\alpha}_1\omega_1 + s(\overline{\alpha}_1)F_{FRM}).$$

Since we have $\omega_0 + \overline{\alpha}_1\omega_1 - s(\overline{\alpha}_1)F_{FRM} > c_D\mathbf{1'K_D^1} + F_D$, it follows that $\frac{\partial}{\partial a}\Delta_F < \frac{\partial}{\partial a}\Delta_D$. This concludes the proof for the robustness result with respect to capital market condition.
Robustness with respect to product market conditions (ρ, σ). We only provide the proof for ρ. From Lemma 6, we have $\frac{\partial}{\partial \rho}S^{FRM} \leq 0$ and $\frac{\partial}{\partial \rho}S^{-FRM} \leq 0$. Therefore, it is sufficient to show $\frac{\partial}{\partial \rho}S^{FRM} \geq \frac{\partial}{\partial \rho}S^{FRM}$ to prove the result of higher robustness for small firms. It follows from (5.39) that this condition is equivalent to $\frac{\partial}{\partial a}\Delta_F \geq 0$. The result follows from Proposition 7. ∎ **Proof of Proposition 10:** To demonstrate the ambiguous effect of financial risk management on expected (stage 0) capacity investment level, it is sufficient to provide examples for each case of $\mathbf{E}[\mathbf{1'K_{T^*}^*}(B_{-FRM}(\alpha_1))] \gtreqless \mathbf{E}[\mathbf{1'K_{T^*}^*}(B_{FRM}(\alpha_1, H_{T^*}^*))]$. We consider $F_F = F_D = 0$ which implies from Proposition 3 that the firm optimally fully

hedges with both technologies ($\Omega_T^4 = \emptyset$). Let $F_{FRM} = 0$ such that financial risk management is costless. Without loss of generality we consider $c_F < \bar{c}_F$ which implies from Proposition 4 that $T^* = F$ with or without financial risk management. Let E be sufficiently large ($E \geq \frac{c_T \mathbf{1}'\mathbf{K_T^1}(1-ab)}{-(b+1)a}$ is sufficient as follows from Lemma 9 in Appendix 5) such that the firm does not borrow up to the credit limit ($\Omega_F^3 = \emptyset$). With these parameter restrictions, we obtain

$$\mathbf{E}[\mathbf{K_F^*}(B_{-FRM}(\alpha_1))] = \int_{\Omega_F^0} \mathbf{K_F^0}\ dR_{B_{-FRM}}(\tilde{B}) + \int_{\Omega_F^1} \overline{\mathbf{K}}_\mathbf{F}\ dR_{B_{-FRM}}(\tilde{B}) + \int_{\Omega_T^2} \mathbf{K_F^1}\ dR_{B_{-FRM}}(\tilde{B}),$$

$$\mathbf{E}[\mathbf{K_F^*}(B_{FRM}(\alpha_1,\omega_1))] = \begin{cases} \mathbf{K_F^0} & \text{if } \omega_0 + \overline{\alpha}_1\omega_1 \in \Omega_F^0 \\ \overline{\mathbf{K}}_\mathbf{F} & \text{if } \omega_0 + \overline{\alpha}_1\omega_1 \in \Omega_F^1 \\ \mathbf{K_F^1} & \text{if } \omega_0 + \overline{\alpha}_1\omega_1 \in \Omega_F^2. \end{cases}$$

We have $\mathbf{K_F^0} > \mathbf{K_F^1}$, and $\mathbf{K_F^0} > \overline{\mathbf{K}}_\mathbf{F} \geq \mathbf{K_F^1}$ for $\tilde{B} \in \Omega_F^1$ with equality only holding for the lower bound of the region Ω_F^1. For $\omega_0 \in \Omega_F^0$ (and hence $\omega_0 + \overline{\alpha}_1\omega_1 \in \Omega_F^0$), $\mathbf{E}[\mathbf{K_F^*}(B_{-FRM}(\alpha_1))] = \mathbf{E}[\mathbf{K_F^*}(B_{FRM}(\alpha_1,\omega_1))]$. For $\omega_0 \in \Omega_F^2$ and $\omega_0+\overline{\alpha}_1\omega_1 \in \Omega_F^0$, $\mathbf{E}[\mathbf{K_F^*}(B_{-FRM}(\alpha_1))] < \mathbf{E}[\mathbf{K_F^*}(B_{FRM}(\alpha_1,\omega_1))]$. For $\omega_0 + \overline{\alpha}_1\omega_1 \in \Omega_F^2$ (and hence $\omega_0 \in \Omega_F^2$), $\mathbf{E}[\mathbf{K_F^*}(B_{-FRM}(\alpha_1))] > \mathbf{E}[\mathbf{K_F^*}(B_{FRM}(\alpha_1,\omega_1))]$.

If we relax our assumption on E, we obtain

$$\mathbf{E}[e_F^*(B_{-FRM}(\alpha_1))] = \int_{\Omega_F^2} [\mathbf{c_F K_F^1} - \tilde{B}]\ dR_{B_{-FRM}}(\tilde{B}) + \int_{\Omega_F^3} E\ dR_{B_{-FRM}}(\tilde{B}),$$

$$\mathbf{E}[e_F^*(B_{FRM}(\alpha_1,\omega_1))] = \begin{cases} 0 & \text{if } \omega_0 + \overline{\alpha}_1\omega_1 \in \Omega_F^{01} \\ \mathbf{c_F K_F^1} - \omega_0 - \overline{\alpha}_1\omega_1 & \text{if } \omega_0 + \overline{\alpha}_1\omega_1 \in \Omega_F^2 \\ E & \text{if } \omega_0 + \overline{\alpha}_1\omega_1 \in \Omega_F^3. \end{cases}$$

It follows that for $\omega_0 + \overline{\alpha}_1\omega_1 \in \Omega_F^3$ we have $\mathbf{E}[e_F^*(B_{-FRM}(\alpha_1))] < \mathbf{E}[e_F^*(B_{FRM}(\alpha_1,\omega_1))]$ and for $\omega_0 + \overline{\alpha}_1\omega_1 \in \Omega_F^0$ we have $\mathbf{E}[e_F^*(B_{-FRM}(\alpha_1))] > \mathbf{E}[e_F^*(B_{FRM}(\alpha_1,\omega_1))]$. ■

Proof of Corollary 5: The proof follows from Proposition 8. ■

Proof of Corollary 6: From Proposition 3, it follows that small firms, as we define in §2.7, optimally fully speculates $H_T^* = -\frac{\omega_0}{\overline{\alpha}_1}$. For $\omega_0 = 0$, the firm optimally does not engage in financial risk management. The low value of integration follows from a continuity argument and the bounded derivative of expected (stage 0) equity value with respect to ω_0. For large firms, financial risk management does not have any value if $\omega_0 \geq \mathbf{c_T 1'K_T^0} + F_T$, i.e. the cash level is sufficient to finance the budget-unconstrained optimal capacity investment level. Low value of financial risk management at high cash levels follow from similar arguments with small firms. When the firm uses financial risk management only for hedging purposes, it follows from Proposition 5 that small firms optimally do not engage in financial risk management. Therefore, the value of integration is zero for small firms. Large firms tend to use financial risk management for full-hedging purposes. For $\omega_0 < \mathbf{c_T 1' K_T^0} + F_T$, financial risk management has positive value; hence the value of integration is higher for large firms than small firms. This concludes the proof. ■

Proposition 29 *If the firm does not engage in financial risk management, there exists a unique technology fixed cost threshold* $\underline{F}_T^{-FRM} < \frac{c_T \mathbf{1'K_T^0}}{-(b+1)(1-\gamma_T)}$ *for technology* $T \in \{D, F\}$ *such that when* $F_T < \underline{F}_T^{-FRM}$*, investing in technology* T *without financial risk management is more profitable than not investing in technology.*

If the firm engages in financial risk management, only one of the following cases holds, depending on the level of the fixed cost F_{FRM}*:*

i) There exists a unique technology fixed cost threshold $\underline{F}_T^{FRM} \leq \frac{\frac{c_T \mathbf{1'K_T^0}}{-(b+1)} - \left(\beta + \frac{(1-\beta)\overline{\alpha}_1}{\alpha_0}\right) F_{FRM}}{1-\gamma_T}$ *for technology* $T \in \{D, F\}$ *such that when* $F_T < \underline{F}_T^{FRM}$*, investing in technology* T *is more profitable than not investing in technology; this case occurs at sufficiently low levels of* F_{FRM}*.*

ii) Not investing in technology is more profitable for $F_T \geq 0$*.*

Proof of Proposition 29: We first prove the first part of the proposition. From Lemma 6 in the proof of Proposition 4 (using $H = 0$ and $F_{FRM} = 0$), $\mathbf{E}\left[\pi_T(B_{-FRM}(\alpha_1), F_T)\right]$ is strictly decreasing in F_T. We define $L_T(\tilde{B}) \doteq \pi_T(\tilde{B}) - (\tilde{B} + P)$, the difference between the equity values of investing in technology T and not investing in technology at each state $\tilde{B}$. It is easy to verify that for $F_T^0 \doteq 0$, $\pi_T(\tilde{B}) > \tilde{B} + P$ for $\tilde{B} \geq 0$. It follows that $\mathbf{E}\left[\pi_T(F_T^0, B_{-FRM}(\alpha_1))\right] > \omega_0 + \overline{\alpha}_1\omega_1 + P$. For $F_T^1 > \frac{c_T \mathbf{1'K_T^0}}{-(b+1)(1-\gamma_T)}$ we have $L_T(\tilde{B}) < 0$ for $\tilde{B} \geq 0$. It follows that $\mathbf{E}\left[\pi_T(F_T^1, B_{-FRM}(\alpha_1))\right] < \omega_0 + \overline{\alpha}_1\omega_1 + P$. Since $\mathbf{E}\left[\pi_T(F_T, B_{-FRM}(\alpha_1))\right]$ is strictly decreasing in F_T, there exists a unique $\underline{F}_T^{-FRM} < \frac{c_T \mathbf{1'K_T^0}}{-(b+1)(1-\gamma_T)}$.

The second part of the proposition follows from a similar argument. We obtain $\mathbf{E}\left[\pi_T(B_{FRM}(\alpha_1, H_T^*))\right] \leq \omega_0 + \overline{\alpha}_1\omega_1 - \left(\beta + \frac{(1-\beta)\overline{\alpha}_1}{\alpha_0}\right) F_{FRM} - (1-\gamma_T)F_T + \frac{c_T \mathbf{1'K_T^0}}{-(b+1)} + P$, where the latter is the expected (stage 0) equity value with budget-unconstrained optimal capacity investment. It follows that for $F_T > F_T^1 = \frac{\frac{c_T \mathbf{1'K_T^0}}{-(b+1)} - \left(\beta + \frac{(1-\beta)\overline{\alpha}_1}{\alpha_0}\right) F_{FRM}}{1-\gamma_T}$, not investing in technology is more profitable.

Two cases may arise with respect to the level of F_{FRM}. When F_{FRM} is sufficiently low, for $F_T^0 = 0$ we have $\mathbf{E}\left[\pi_T(F_T^0, B_{FRM}(\alpha_1, H_T^*))\right] > \omega_0 + \overline{\alpha}_1\omega_1 + P$. In this case (case i), a unique $\underline{F}_T^{FRM} < F_T^1$ exists since $\mathbf{E}\left[\pi_T(F_T, B_{FRM}(\alpha_1, H_T^*))\right]$ is strictly decreasing in F_T. For a sufficiently high level of F_{FRM} and appropriate allocation scheme β (that makes such a F_{FRM} feasible), not investing in technology is more profitable for $F_T = 0$. In this case (case ii), $\underline{F}_T^{FRM}$ does not exist and not investing in technology is more profitable for $F_T \geq 0$. ■

Proposition 30 *Only one of the following cases holds for technology T:*

i) There exists a unique financial risk management fixed cost threshold $\underline{F}_{FRM}^T$ such that when $F_{FRM} < \underline{F}_{FRM}^T$, it is more profitable to engage in financial risk management than not;

ii) For any feasible F_{FRM}, engaging in financial risk management is more profitable than not.

Proof of Proposition 30: The proof follows from showing that $\mathbf{E}\left[\pi_T\left(B_{FRM}(\alpha_1, H_T^*)\right)\right]$ strictly decreases in F_{FRM}. From Lemma 2, we can interchange the derivative and expectation operators and using the Leibniz' rule we obtain

$$\begin{aligned}
\mathbf{E}\left[\frac{\partial \pi_T(B_{FRM}(\alpha_1, H))}{\partial F_{FRM}}\right] &= \int_{\max(\alpha_T^0,0)}^{\infty} (-\beta - \frac{1-\beta}{\alpha_0}x)\, r_{\alpha_1}(x)\, dx \\
&+ \int_{\max(\alpha_T^1,0)}^{\max(\alpha_T^0,0)} \frac{M_T(1+1/b)}{c_T}\left(\frac{U(x)}{c_T}\right)^{\frac{1}{b}} (-\beta - \frac{1-\beta}{\alpha_0}x)\, r_{\alpha_1}(x)\, dx \\
&+ \int_{\max(\alpha_T^2,0,\alpha_T^B)}^{\max(\alpha_T^1,0)} (-\beta - \frac{1-\beta}{\alpha_0}x)(1+a)\, r_{\alpha_1}(x)\, dx \\
&+ \int_{\max(0,\alpha_T^B)}^{\max(\alpha_T^2,0,\alpha_T^B)} \frac{M_T(1+1/b)}{c_T}\left(\frac{U(x)+E}{c_T}\right)^{\frac{1}{b}} (-\beta - \frac{1-\beta}{\alpha_0}x)\, r_{\alpha_1}(x)\, dx \\
&+ \int_{0}^{\max(0,\alpha_T^B)} (-\beta - \frac{1-\beta}{\alpha_0}x)\, r_{\alpha_1}(x)\, dx
\end{aligned} \tag{5.41}$$

for any feasible H, where $U(x) = \omega_0^{FRM} + x(\omega_1^{FRM} - H) + \overline{\alpha}_1 H - F_T$. Since all terms are negative, it follows that $\mathbf{E}\left[\pi_T\left(B_{FRM}(\alpha_1, H_T^*)\right)\right]$ is strictly decreasing in F_{FRM}. For $F_{FRM} = 0$, we have $\mathbf{E}\left[\pi_T\left(B_{FRM}(\alpha_1, H_T^*)\right)\right] \geq \mathbf{E}\left[\pi_T(B_{-FRM}(\alpha_1))\right]$ from the optimality of H_T^*. The existence of $\underline{F}_{FRM}^T \leq \min\left(\frac{\omega_0}{\beta}, \frac{\alpha_0\omega_1}{1-\beta}\right)$ depends on the allocation scheme β. If β is such that a sufficiently large level of F_{FRM} is feasible, then since $\mathbf{E}\left[\pi_T\left(B_{FRM}(\alpha_1, H_T^*)\right)\right]$ is strictly decreasing in F_{FRM}, there exists a unique $\underline{F}_{FRM}^T$ (case i). Otherwise, since $\mathbf{E}\left[\pi_T\left(B_{FRM}(\alpha_1, H_T^*)\right)\right]$ is preferred for $F_{FRM} = 0$, case ii holds.

We show that $\exists\, \beta$ such that case i holds. Let $\beta = \frac{\omega_0}{\omega_0+\alpha_0\omega_1}$. It follows that the condition $F_{FRM} \leq \min\left(\frac{\omega_0}{\beta}, \frac{\alpha_0\omega_1}{1-\beta}\right)$ is equivalent to $F_{FRM} \leq \omega_0+\alpha_0\omega_1$. We obtain $\lim_{F_{FRM}\to\omega_0+\alpha_0\omega_1} B_{FRM}(\alpha_1, H) = 0$; therefore $\mathbf{E}\left[\pi_T\left(B_{FRM}(\alpha_1, H_T^*)\right)\right]|_{F_{FRM}\to\omega_0+\alpha_0\omega_1} < \mathbf{E}\left[\pi_T(B_{-FRM}(\alpha_1))\right]$. It follows that a unique $\underline{F}_{FRM}^T$ exists. ■

Proposition 31 *For technology $T \in \{D, F\}$ there exists a unique variable cost threshold $\overline{c}_T(c_{-T}, H_T^*, 0)$ such that investing in technology T with financial risk management is more profitable than investing in the other technology $(-T)$ without financial risk management.*

Proof of Proposition 31: The proof follows as in Proposition 4, and is omitted. ■

Appendix B. Characterization of $\widehat{B}_T$

Recall from Proposition 2 that $\widehat{B}_T$ is the budget threshold below which the firm does not borrow or invest. From the proof of Proposition 2, for $F_T \geq E$, $\widehat{B}_T > F_T - E$ is the unique solution to $G_T(\widehat{B}_T) = 0$ where $G_T(\tilde{B}) \doteq \Psi_T(\tilde{B}) - (\tilde{B} - (1-\gamma_T)F_T + P)$, the difference between the equity values in (5.24) and not borrowing and not investing in capacity. For $F_T < E$, $\widehat{B}_T$, if it exists on $[0, \infty)$, is unique. For notational convenience, we let $\widehat{B}_T \doteq 0$ if the two curves do not intersect on the domain of $G_T(.)$ for $F_T < E$. From (5.24) for $\tilde{B} \geq F_T$ we obtain $\lim_{\mathbf{K_T} \to \mathbf{0}^+} \nabla_{\mathbf{K_T}} \Psi_T \to \infty$. It follows that the firm always optimally invests in capacity if internal budget $\tilde{B}$ is sufficient to cover the fixed cost of the technology. We conclude that $F_T - E < \widehat{B}_T < F_T$. Since $\Psi_T(\tilde{B})$ can take four different forms we have four different cases to analyze.

Case 1: $c_T \mathbf{1}'\mathbf{K_T^0} + F_T \leq \tilde{B}$

From (5.28), $G_T(\tilde{B}) > 0$ in this range, so it is not possible to have $c_T \mathbf{1}'\mathbf{K_T^0} + F_T \leq \widehat{B}_T$.

Case 2: $c_T \mathbf{1}'\mathbf{K_T^1} + F_T \leq \tilde{B} < c_T \mathbf{1}'\mathbf{K_T^0} + F_T$

$$\begin{aligned} G_T(\tilde{B}) &= M_T \frac{(\tilde{B} - F_T)}{c_T} \left(\frac{c_T}{\tilde{B} - F_T} \right)^{-\frac{1}{b}} + \gamma_T F_T + P - (\tilde{B} - (1-\gamma_T)F_T + P) \\ &\geq M_T \frac{(\tilde{B} - F_T)}{c_T} \left(\frac{1}{\mathbf{1}'\mathbf{K_T^0}} \right)^{-\frac{1}{b}} + F_T - \tilde{B} = \frac{-1}{b+1}(\tilde{B} - F_T) > 0. \end{aligned}$$

Therefore, it is not possible to have $c_T \mathbf{1}'\mathbf{K_T^1} + F_T \leq \widehat{B}_T < c_T \mathbf{1}'\mathbf{K_T^0} + F_T$.

Case 3: $c_T \mathbf{1}'\mathbf{K_T^1} + F_T - E \leq \tilde{B} < c_T \mathbf{1}'\mathbf{K_T^1} + F_T$

$$\begin{aligned} G_T(\widehat{B}_T) &= (\widehat{B}_T - F_T)(1+a) + \frac{c_T \mathbf{1}'\mathbf{K_T^1}(1+a)}{-(b+1)} + \gamma_T F_T + P - (\widehat{B}_T - (1-\gamma_T)F_T + P) = 0 \\ &\Rightarrow \widehat{B}_T = F_T - \frac{c_T \mathbf{1}'\mathbf{K_T^1}(1+a)}{-(b+1)a}. \end{aligned}$$

For $\widehat{B}_T$ to be feasible in Case 3, $\widehat{B}_T \geq 0$ and $\widehat{B}_T \geq c_T \mathbf{1}'\mathbf{K_T^1} + F_T - E$ should hold. Therefore, if $F_T \geq \frac{c_T \mathbf{1}'\mathbf{K_T^1}(1+a)}{-(b+1)a}$ and $E \geq \frac{c_T \mathbf{1}'\mathbf{K_T^1}(1-ab)}{-(b+1)a}$ then $\widehat{B}_T$ is feasible. Otherwise, it is not possible to have $c_T \mathbf{1}'\mathbf{K_T^1} + F_T - E \leq \widehat{B}_T < c_T \mathbf{1}'\mathbf{K_T^1} + F_T$.

Case 4: $c_T \mathbf{1}'\mathbf{K_T^1} + F_T - E > \tilde{B}$

In this case, we can derive a sufficient condition for non-existence of intersection. We obtain

$$\begin{aligned} G_T(\tilde{B}) &= -E(1+a) + M_T \frac{(E+\tilde{B}-F_T)}{c_T}\left(\frac{c_T}{E+\tilde{B}-F_T}\right)^{-\frac{1}{b}} + \gamma_T F_T + P - (\tilde{B} - (1-\gamma_T)F_T + P) \\ &\geq -E(1+a) + M_T \frac{(E+\tilde{B}-F_T)}{c_T}\left(\frac{1}{\mathbf{1}'\mathbf{K}_{\mathbf{T}}^{\mathbf{1}}}\right)^{-\frac{1}{b}} + F_T - \tilde{B} \\ &\geq \frac{E(1+a)}{-(b+1)} + \frac{1-ab}{-(b+1)}(\tilde{B} - F_T). \end{aligned}$$

Therefore if $F_T < \frac{E(1+a)}{1-ab}$, then $G_T(\tilde{B}) > 0$ and it is not possible to have $c_T\mathbf{1}'\mathbf{K}_{\mathbf{T}}^{\mathbf{1}} + F_T - E > \widehat{B}_T$. Otherwise, $\widehat{B}_T$ is a solution of a non-integer polynomial of degree $\frac{b}{b+1}$ and it is not possible to find closed-form expression in the whole range of parameters. The following lemma summarizes the analysis and provides a closed-form expression for $\widehat{B}_T$ for a subset of parameter levels.

Lemma 9 *Let E be such that $E \geq \frac{c_T\mathbf{1}'\mathbf{K}_{\mathbf{T}}^{\mathbf{1}}(1-ab)}{-(b+1)a}$.*
If $F_T \leq \frac{c_T\mathbf{K}_{\mathbf{T}}^{\mathbf{1}}(1+a)}{-(b+1)a}$ then $\widehat{B}_T = 0$ and $\Omega_T^{34} = \emptyset$.
If $F_T > \frac{c_T\mathbf{1}'\mathbf{K}_{\mathbf{T}}^{\mathbf{1}}(1+a)}{-(b+1)a}$ then $\widehat{B}_T = F_T - \frac{c_T\mathbf{1}'\mathbf{K}_{\mathbf{T}}^{\mathbf{1}}(1+a)}{-(b+1)a}$ and $\Omega_T^3 = \emptyset$.

We also provide the following lemma which we will occasionally use in the comparative statics analysis throughout the paper.

Lemma 10 *The budget threshold $\widehat{B}_T$ is increasing in c_T, F_T, a and decreasing in E.*

Proof We only provide the proof for the result related to a. The other results can be shown in a similar fashion. Let $\widehat{B}_T(a^i)$, $i = 0, 1$ define the threshold levels for an arbitrary $a^0 < a^1$. We want to show that $\widehat{B}_T(a^0) \leq \widehat{B}_T(a^1)$. Notice that not only the functional form of $G_T(\tilde{B})$ in any region but also the budget levels defining the regions in (5.24) depend on a. We obtain

$$\frac{\partial G_T(\tilde{B})}{\partial a} = \begin{cases} 0 & \text{if } c_T\mathbf{1}'\mathbf{K}_{\mathbf{T}}^{\mathbf{0}} + F_T \leq \tilde{B} \\ 0 & \text{if } c_T\mathbf{1}'\mathbf{K}_{\mathbf{T}}^{\mathbf{1}} + F_T \leq \tilde{B} < c_T\mathbf{1}'\mathbf{K}_{\mathbf{T}}^{\mathbf{0}} + F_T \\ \tilde{B} - c_T\mathbf{1}'\mathbf{K}_{\mathbf{T}}^{\mathbf{1}} - F_T & \text{if } c_T\mathbf{1}'\mathbf{K}_{\mathbf{T}}^{\mathbf{1}} + F_T - E \leq \tilde{B} < c_T\mathbf{1}'\mathbf{K}_{\mathbf{T}}^{\mathbf{1}} + F_T \\ -E & \text{if } F_T - E < \tilde{B} < c_T\mathbf{1}'\mathbf{K}_{\mathbf{T}}^{\mathbf{1}} + F_T - E. \end{cases}$$

at the points where $G_T(\tilde{B})$ is differentiable in a. It follows that $\frac{\partial}{\partial a}G_T(\tilde{B}) \leq 0$ for any $\tilde{B}$ where the function is differentiable. Since $G_T(\tilde{B})$ is a continuous function of $\tilde{B}$ for any a, we conclude that $G_T(\tilde{B})$ is decreasing in a. This implies $G_T(\tilde{B}, a^0) \geq G_T(\tilde{B}, a^1)$

for $\tilde{B} > F_T - E$. At this point, two different cases may arise regarding the definition of $\widehat{B}_T(a^0)$. If $\widehat{B}_T(a^0)$ is the solution of $G_T(\tilde{B}, a^0) = 0$, then we have $G_T(\widehat{B}_T(a^0), a^1) \leq G_T(\widehat{B}_T(a^0), a^0) = 0$. Since $G_T(\tilde{B})$ is increasing in $\tilde{B}$ from (5.25), it follows that $\widehat{B}_T(a^1) \geq \widehat{B}_T(a^0)$. If $\widehat{B}_T(a^0) = 0$ because $G_T(\tilde{B}, a^0) > 0$ for $\tilde{B} \geq 0$, then from (5.25) either we have $\widehat{B_T}(a^1) = 0$, $(G_T(\tilde{B}, a^1) > 0$ for $\tilde{B} \geq 0)$ or $\widehat{B}_T(a^1)$ is a solution to $G_T(\tilde{B}, a^1) = 0$. In either case, we have $\widehat{B}_T(a^1) \geq \widehat{B}_T(a^0)$. ∎

Appendix C. Proofs of Supporting Lemmas

Proof of Lemma 1: From Appendix 5, we calculate

$$\frac{\partial G_T(\tilde{B})}{\partial F_T} = \begin{cases} 0 & \text{if } c_T \mathbf{1}'\mathbf{K_T^0} + F_T \le \tilde{B} \\ -\frac{M_T}{c_T}(1+\frac{1}{b})\left(\frac{\tilde{B}-F_T}{c_T}\right)^{\frac{1}{b}} + 1 & \text{if } c_T \mathbf{1}'\mathbf{K_T^1} + F_T \le \tilde{B} < c_T \mathbf{1}'\mathbf{K_T^0} + F_T \\ -a & \text{if } c_T \mathbf{1}'\mathbf{K_T^1} + F_T - E \le \tilde{B} < c_T \mathbf{1}'\mathbf{K_T^1} + F_T \\ -\frac{M_T}{c_T}(1+\frac{1}{b})\left(\frac{E+\tilde{B}-F_T}{c_T}\right)^{\frac{1}{b}} + 1 & \text{if } F_T - E < \tilde{B} < c_T \mathbf{1}'\mathbf{K_T^1} + F_T - E. \end{cases}$$

From (5.26), (5.27) and the continuity of $G_T(\tilde{B})$, it follows that $G_T(\tilde{B})$ strictly decreases in F_T for $\tilde{B} < c_T\mathbf{1}'\mathbf{K_T^0} + F_T$. Recall from Proposition 2 (or Appendix 5) that either $\widehat{B}_T$ is a solution to $G_T(\tilde{B}) = 0$ or $\widehat{B}_T = 0$ (if $G_T(\tilde{B}) > 0$ for $\tilde{B} \ge 0$).

We first prove the necessity of the second argument. Let $\overline{F}_T$ be the fixed cost that satisfies $G_T(\widehat{B}_T(\overline{F}_T), \overline{F}_T) = 0$ with $\widehat{B}_T(\overline{F}_T) = 0$. In other words, $\overline{F}_T$ is the fixed cost of technology T that makes the two equity values intersect at $\tilde{B} = 0$. From Appendix 5, it follows that for $F_T = 0$, $G_T(\tilde{B}) > 0$ for $\tilde{B} \ge 0$. For $F_T \ge E$, we have $\lim_{\tilde{B}\to(F_T-E)^+} G_T(\tilde{B}) < 0$, and two curves intersect at $\widehat{B}_T > F_T - E$. Since $G_T(\tilde{B})$ is strictly decreasing in F_T, such an $\overline{F}_T < E$ always exists. Let $F_T^0 \le \overline{F}_T$ be an arbitrary fixed cost. We have $G_T(\widehat{B}_T(\overline{F}_T), F_T^0) < G_T(\widehat{B}_T(\overline{F}_T), \underline{F}_T) = 0$ since $\widehat{B}_T < c_T\mathbf{1}'\mathbf{K_T^0} + F_T$ (follows from Appendix 5) and G_T strictly decreases in F_T. From (5.25) we have $G_T(\tilde{B})$ is strictly increasing in $\tilde{B}$ so it follows that $\widehat{B}_T(F_T^0) > \widehat{B}_T(\overline{F}_T) = 0$.

We now prove the necessity of the first argument. Let $F_T^1 < \overline{F}_T$ be an arbitrary fixed cost. Since $\widehat{B}_T(\overline{F}_T) = 0$ and $G_T(\tilde{B})$ strictly decreases in F_T, we have $G_T(\tilde{B}, F_T^1) > 0$ for $\tilde{B} \ge 0$. This implies that $\widehat{B}_T(F_T) = 0$ for $F_T < \overline{F}_T$. The uniqueness of $\overline{F}_T$ follows from the fact that $G_T(\tilde{B})$ is strictly decreasing in F_T and the uniqueness of $\widehat{B}_T$.

The proof for sufficiency follows easily using a contrapositive argument. ■

Proof of Lemma 2: The expectation and differentiation operators can be interchanged if the function under expectation is integrable and satisfies the Lipschitz condition of order one (Glasserman 1994, p.245). The function $\pi_T(\tilde{\alpha}_1)$ satisfies the Lipschitz condition of order one if

$$\frac{|\pi_T(\tilde{\alpha}_1') - \pi_T(\tilde{\alpha}_1'')|}{|\tilde{\alpha}_1' - \tilde{\alpha}_1''|} \le Y_{\pi_T} \;\; \forall\, (\tilde{\alpha}_1', \tilde{\alpha}_1'') > 0 \text{ for some } Y_{\pi_T} \text{ with } \mathbf{E}[Y_{\pi_T}] < \infty. \tag{5.42}$$

Clearly, condition (5.42) is satisfied if $\left|\frac{\partial \pi_T}{\partial \tilde{\alpha}_1}\right|$ is bounded. Note that $\frac{\partial}{\partial \tilde{\alpha}_1}\pi_T = \left(\frac{\partial}{\partial \tilde{B}}\pi_T\right)\left(\frac{\partial}{\partial \tilde{\alpha}_1}\tilde{B}\right) = \left(\frac{\partial}{\partial \tilde{B}}\pi_T\right)(\omega_1 - H_T)$. From Corollary 1, we know that π_T is differentiable in $\tilde{\alpha}_1$ everywhere except at α_T^B as defined in (5.33). If $\tilde{B} \in \Omega_T^1$ we have

$$\frac{\partial \pi_T}{\partial \tilde{B}} \leq \frac{M_T}{c_T}(1+\frac{1}{b})\left(\mathbf{1}'\mathbf{K_T^1}\right)^{\frac{1}{b}} \leq (1+a),$$

and for $\tilde{B} \in \Omega_T^3$ since $\widehat{B_T} \geq 0$ and $E_T > F_T$ (from (5.6)) we have

$$\frac{\partial \pi_T}{\partial \tilde{B}} \leq \frac{M_T}{c_T}(1+\frac{1}{b})\left(\frac{E-F_T}{c_T}\right)^{\frac{1}{b}} \leq Y_T$$

where $1 + a_T < Y_T < \infty$. It follows that $\left|\frac{\partial \pi_T}{\partial \tilde{\alpha}_1}\right| \leq Y_T(\omega_1 - H_T) < \infty$ for $\alpha_1 \geq 0$ except α_T^B. Since π_T is continuous in α_1 and the first derivative is bounded at the differentiable points of π_T, the non-differentiability at α_T^B does not violate (5.42). Since $\pi_T(\tilde{\alpha}_1)$ is integrable, the interchange of the derivative and expectation is justified. ■

Proof of Lemma 3: From Lemma 2, we can interchange the derivative and the expectation operators and using the Leibniz' rule we obtain

$$\begin{aligned}\frac{\partial \mathbf{E}[\pi_T(B_{FRM}(\alpha_1,H))]}{\partial c_T} &= \int_{\Omega_T^0} -\mathbf{1}'\mathbf{K_T^0}\ dR_{B_{FRM}(H)}(\tilde{B}) \\ &+ \int_{\Omega_T^1} -(1+\frac{1}{b})\frac{M_T}{c_T}\left(\frac{\tilde{B}-F_T}{c_T}\right)^{1+\frac{1}{b}} dR_{B_{FRM}(H)}(\tilde{B}) \\ &+ \int_{\Omega_T^2} -\mathbf{1}'\mathbf{K_T^1}(1+a)\ dR_{B_{FRM}(H)}(\tilde{B}) \\ &+ \int_{\Omega_T^3} -(1+\frac{1}{b})\frac{M_T}{c_T}\left(\frac{E+\tilde{B}-F_T}{c_T}\right)^{1+\frac{1}{b}} dR_{B_{FRM}(H)}(\tilde{B}).\end{aligned} \tag{5.43}$$

It follows that $\frac{\partial}{\partial c_T}\mathbf{E}[\pi_T(B_{FRM}(\alpha_1,H))] \leq 0$ with equality holding only for $H = \omega_1^{FRM}$ and $\omega_0^{FRM} + \overline{\alpha}_1\omega_1^{FRM} \in \Omega_T^4$ (i.e. $\Omega_T^{0123} = \emptyset$). From Proposition 3, we know that in this case $H_T^* = -\frac{\omega_0^{FRM}}{\overline{\alpha}_1}$, so we can ignore $H = \omega_1^{FRM}$. In other words, in the relevant set of $B_{FRM}(\alpha_1, H)$ we have $\frac{\partial}{\partial c_T}\mathbf{E}[\pi_T(B_{FRM}(\alpha_1,H))] < 0$. ■

Proof of Lemma 4:

Case i): The proof follows from Lemma 3 of Chod et al. (2006) by substituting $\tau = 1$ and noting that ρ and σ in that paper correspond to parameters of the underlying bivariate normal distribution ($\ln\boldsymbol{\xi}$) of $\boldsymbol{\xi}$. In our paper, ρ and σ are the parameters of $\boldsymbol{\xi}$ in the covariance matrix $\boldsymbol{\Sigma}$.

Case ii): We only prove the more general case where both of the marginal distributions of $\boldsymbol{\xi}$ are pairwise stochastically more variable than the marginal distributions of $\boldsymbol{\xi}$. The proof for the case where one of the marginals is identical for ξ_i' and ξ_i is a special case of this proof. For $\xi_i \geq 0$, $\xi_i^{'} \geq 0$ and $\overline{\xi}_i = \overline{\xi'}_i$ it follows from Ross (1983, p.271) that $\xi_i^{'} \succeq_v \xi_i$ if and only if $\mathbf{E}[h(\xi_i)] \leq \mathbf{E}[h(\xi_i^{'})]$ for all convex functions $h(.)$. With independent marginal distributions of $\boldsymbol{\xi}$ we have

$$\mathbf{E}\left[\left(\xi_1^{-b}+\xi_2^{-b}\right)^{-\frac{1}{b}}\right] = \int_0^{\infty}\int_0^{\infty}\left(x_1^{-b}+x_2^{-b}\right)^{-\frac{1}{b}} f_1(x_1)f_2(x_2)dx_1dx_2 = \int_0^{\infty} g(x_1;x_2)f_1(x_1)dx_1$$

where $f_i(.)$ is the marginal distribution of ξ_i and $g(k;x_2) = \int_0^{\infty}\left(k^{-b}+x_2^{-b}\right)^{-\frac{1}{b}} f_2(x_2)dx_2$ for $k \geq 0$. To conclude the proof, we need to show that $g(k;x_2)$ is convex in k and $\left(k^{-b}+x_2^{-b}\right)^{-\frac{1}{b}}$ is convex in x_2. To prove both of the desired convexity results, it is sufficient to show that $g^{'}(k,x_2)$ is convex in k. We obtain

$$\frac{\partial^2 g^{'}}{\partial k} = (-b-1)\left(k^{-b}+x_2^{-b}\right)^{-\frac{1}{b}-1} k^{-b-2}\frac{x_2^{-b}}{\left(k^{-b}+x_2^{-b}\right)} \geq 0$$

for $k \geq 0$ and $x_2 \geq 0$. This concludes the proof.

Case *iii*): Follows from (5.35) in the proof of Proposition 4. ■

Proof of Lemma 5:

Case *i*): The proof follows from Lemma 4 of Chod et al. (2006) by substituting $\tau = 1$ and noting that ρ and σ in that paper correspond to parameters of the underlying bivariate normal distribution $(\ln \boldsymbol{\xi})$ of $\boldsymbol{\xi}$. In our paper, ρ and σ are the parameters of $\boldsymbol{\xi}$ in the covariance matrix $\boldsymbol{\Sigma}$.

Case *ii*): The proof of this case is adapted from Corbett and Rajaram (2005). If $\boldsymbol{\xi} \succeq_c \boldsymbol{\xi}$, it follows from Muller and Scarsini (2000, p.110) that $\boldsymbol{\xi} \succeq_{sm} \boldsymbol{\xi}$ ($\boldsymbol{\xi}$ dominates $\boldsymbol{\xi}$ in the sense of supermodular order). From the definition of supermodular stochastic ordering, it is sufficient to show that $g(\xi_1,\xi_2) = -\left(\xi_1^{-b}+\xi_2^{-b}\right)^{-\frac{1}{b}}$ is supermodular. From Muller and Scarcini (2003), it follows that g is supermodular if and only if all mixed derivatives are non-negative, i.e. $\frac{\partial^2}{\partial\xi_1\partial\xi_2} g \geq 0$ for $\boldsymbol{\xi} \geq \mathbf{0}$. We obtain

$$\frac{\partial^2 g}{\partial\xi_1\partial\xi_2} = (-b-1)\left(\xi_1^{-b}+\xi_2^{-b}\right)^{-\frac{1}{b}-2}(\xi_1\xi_2)^{-b-2} \geq 0.$$

This concludes the proof.

Case *iii*): Follows from (5.35) in the proof of Proposition 4. ■

Proof of Lemma 6:

Case *i*): As in Lemma 3, we obtain

$$\begin{aligned}\frac{\partial \mathbf{E}[\pi_T(B_{FRM}(\alpha_1, H))]}{\partial F_T} &= \int_{\Omega_T^0} -(1-\gamma_T)\; dR_{B_{FRM}(H)}(\tilde{B}) \qquad (5.44)\\ &+ \int_{\Omega_T^1} -\left[(1+\frac{1}{b})\frac{M_T}{c_T}\left(\frac{\tilde{B}-F_T}{c_T}\right)^{\frac{1}{b}} - \gamma_T\right] dR_{B_{FRM}(H)}(\tilde{B})\\ &+ \int_{\Omega_T^2} -(1-\gamma_T+a)\; dR_{B_{FRM}(H)}(\tilde{B})\\ &+ \int_{\Omega_T^3} -\left[(1+\frac{1}{b})\frac{M_T}{c_T}\left(\frac{E+\tilde{B}-F_T}{c_T}\right)^{\frac{1}{b}} - \gamma_T\right] dR_{B_{FRM}(H)}(\tilde{B})\\ &+ \int_{\Omega_T^4} -(1-\gamma_T)\; dR_{B_{FRM}(H)}(\tilde{B}).\end{aligned}$$

Since $\gamma_T < 1$ by definition, it follows from (5.26) and (5.27) that the second and the fourth terms are negative. This implies that $\frac{\partial}{\partial F_T}\mathbf{E}[\pi_T(B_{FRM}(\alpha_1, H))] < 0$. We have $\frac{\partial}{\partial \gamma_T}\mathbf{E}[\pi_T(B_{FRM}(\alpha_1, H))] = F_T$ and it follows that the expected (stage 0) equity value is strictly increasing in the salvage rate for $F_T > 0$.

Case *ii*): We obtain

$$\frac{\partial \mathbf{E}[\pi_T(B_{FRM}(\alpha_1, H))]}{\partial E} = \int_{\Omega_T^3} \left(-(1+a) + \frac{M_T\,(1+\frac{1}{b})}{c_T}\left(\frac{E+\tilde{B}-F_T}{c_T}\right)^{\frac{1}{b}}\right)\; dR_{B_{FRM}(H)}(\tilde{B}).$$

It follows that $\frac{\partial}{\partial E}\mathbf{E}[\pi_T(B_{FRM}(\alpha_1, H))] \geq 0$ with equality holding for H such that $\omega_0^{FRM} + \overline{\alpha}_1 H \geq c_T \mathbf{1}'\mathbf{K}_\mathbf{T}^\mathbf{1} + F_T - E$; or $H = \omega_1^{FRM}$ and $\omega_0^{FRM} + \overline{\alpha}_1\omega_1^{FRM} < \widehat{B}_T$. From Proposition 3 we know that in the latter case $H_T^* = -\frac{\omega_0^{FRM}}{\overline{\alpha}_1}$ and we can ignore this case in the relevant set of financial risk management levels.

Case *iii*): We obtain

$$\frac{\partial \mathbf{E}[\pi_T(B_{FRM}(\alpha_1, H))]}{\partial a} = \int_{\Omega_T^2} \left(\tilde{B} - c_T\mathbf{1}'\mathbf{K}_\mathbf{T}^\mathbf{1} - F_T\right)\; dR_{B_{FRM}(H)}(\tilde{B}) - \int_{\Omega_T^3} E_T\; dR_{B_{FRM}(H)}(\tilde{B}).$$

It follows that $\frac{\partial}{\partial a}\mathbf{E}[\pi_T(B_{FRM}(\alpha_1, H))] \leq 0$ with equality holding for $\omega_0^{FRM} + \overline{\alpha}_1 H \geq c_T\mathbf{1}'\mathbf{K}_\mathbf{T}^\mathbf{1} + F_T$; or $H = \omega_1^{FRM}$ and $\omega_0^{FRM} + \overline{\alpha}_1\omega_1^{FRM} < \widehat{B}_T$. From Proposition 3 we know that in the latter case $H_T^* = -\frac{\omega_0^{FRM}}{\overline{\alpha}_1}$ and we can ignore this case in the relevant set of financial risk management levels.

Case *iv*): The expected (stage 0) equity value with dedicated technology is independent of

σ. Therefore, we focus only on flexible technology. We obtain

$$\frac{\partial \mathbf{E}[\pi_F(B_{FRM}(\alpha_1,H))]}{\partial \sigma} = \int_{\Omega_F^0} \frac{\partial M_F}{\partial \sigma}\left(c_F M_F(1+\frac{1}{b})\right)^{-b-1} dR_{B_{FRM}(H)}(\tilde{B}) \quad (5.45)$$
$$+ \int_{\Omega_F^1} \frac{\partial M_F}{\partial \sigma}\left(\frac{\tilde{B}-F_F}{c_F}\right)^{1+\frac{1}{b}} dR_{B_{FRM}(H)}(\tilde{B})$$
$$+ \int_{\Omega_F^2} \frac{\partial M_F}{\partial \sigma}\left(\frac{c_F M_F(1+\frac{1}{b})}{1+a}\right)^{-b-1} dR_{B_{FRM}(H)}(\tilde{B})$$
$$+ \int_{\Omega_F^3} \frac{\partial M_F}{\partial \sigma}\left(\frac{E+\tilde{B}-F_F}{c_F}\right)^{1+\frac{1}{b}} dR_{B_{FRM}(H)}(\tilde{B}).$$

From Lemma 4, we have $\frac{\partial}{\partial \sigma} M_F \geq 0$ with respect to our definitions of demand variability. It follows that $\frac{\partial}{\partial \sigma}\mathbf{E}[\pi_T(B_{FRM}(\alpha_1,H))] \geq 0$.

Case v): The proof of the comparative static result with respect to ρ is similar to σ and is omitted. ■

Proof of Lemma 7: It is easy to verify that we have $c_F \mathbf{K_F^j}\Big|_{\bar{c}_F^S(c_D)} = c_D \mathbf{1}'\mathbf{K_D^j}$ for $j = 0,1$. Since $F_F = F_D$ and $\gamma_F = \gamma_D$ from (5.24) we have $\Psi_F(\tilde{B}) = \Psi_D(\tilde{B})$ which implies $\widehat{B}_F = \widehat{B}_D$. It follows that the regions in (3.1) overlap, i.e. $\Omega_F^i \equiv \Omega_D^i$ for $i = 0,..,4$. Since the budget distribution $B_{FRM}(H)$ is independent of cost parameters, the expected (stage 0) equity values are the same at the threshold level. Moreover, from (5.30), it follows that $H_F^*(\bar{c}_F^S(c_D)) = H_D^*(c_D)$ because both of them are solutions to the same optimization problem. ■

Proof of Lemma 8: Recall from the proof of Proposition 7 we have

$$\Upsilon^{\varphi} \doteq \frac{\partial \Delta_T}{\partial \varphi} = \frac{\partial \mathbf{E}\left[\pi_T\left(B_{FRM}(\alpha_1,\omega_1^{FRM})\right)\right]}{\partial \varphi} - \frac{\partial \mathbf{E}\left[\pi_T(B_{-FRM}(\alpha_1))\right]}{\partial \varphi}$$

For $\varphi = c_T$ ($\varphi = F_T$), we calculate the derivative from Lemma 3 (Lemma 6) by letting $\Omega_T^{34} = \emptyset$ (because of our assumptions on F_T and E).

In (5.43) of Lemma 6, for $\tilde{B} \in \Omega_T^1$ we have

$$1-\gamma_T < \left[(1+\frac{1}{b})\frac{M_T}{c_T}\left(\frac{\tilde{B}-F_T}{c_T}\right)^{\frac{1}{b}} - \gamma_T\right] < 1+a-\gamma_T.$$

For $\omega_0^{FRM} + \overline{\alpha}_1\omega_1^{FRM} \in \Omega_T^0$, it follows that

$$\frac{\partial \mathbf{E}[\pi_T(B_{FRM}(\alpha_1,\omega_1^{FRM}))]}{\partial F_T} = -(1-\gamma_T) \geq \frac{\partial \mathbf{E}[\pi_T(B_{-FRM}(\alpha_1))]}{\partial F_T},$$

and we obtain $\Upsilon^{F_T} \geq 0$ where the equality holds for $\omega_0 > c_T \mathbf{1}'\mathbf{K}^{\mathbf{0}}_{\mathbf{T}} + F_T$.

For $\omega_0^{FRM} + \overline{\alpha}_1 \omega_1^{FRM} \in \Omega_T^2$,

$$\frac{\partial \mathbf{E}[\pi_T(B_{FRM}(\alpha_1, \omega_1^{FRM}))]}{\partial F_T} = -(1 + a - \gamma_T) < \frac{\partial \mathbf{E}[\pi_T(B_{-FRM}(\alpha_1))]}{\partial F_T},$$

and we obtain $\Upsilon^{F_T} < 0$. This concludes the proof for part (i).

Similarly, in (5.44) of Lemma 6, for $\tilde{B} \in \Omega_T^1$ we have

$$\left|\mathbf{1}'\mathbf{K}^{\mathbf{0}}_{\mathbf{T}}\right| > \left|(1 + \frac{1}{b})\frac{M_T}{c_T}\left(\frac{\tilde{B} - F_T}{c_T}\right)^{1+\frac{1}{b}}\right| > \left|\mathbf{1}'\mathbf{K}^{\mathbf{1}}_{\mathbf{T}}(1+a)\right|.$$

For $\omega_0^{FRM} + \overline{\alpha}_1 \omega_1^{FRM} \in \Omega_T^0$, it follows that

$$\frac{\partial \mathbf{E}[\pi_T(B_{FRM}(\alpha_1, \omega_1^{FRM}))]}{\partial c_T} = -\mathbf{1}'\mathbf{K}^{\mathbf{0}}_{\mathbf{T}} \leq \frac{\partial \mathbf{E}[\pi_T(B_{-FRM}(\alpha_1))]}{\partial c_T}$$

and we obtain $\Upsilon^{c_T} \leq 0$ where the equality holds for $\omega_0 > c_T \mathbf{1}'\mathbf{K}^{\mathbf{0}}_{\mathbf{T}} + F_T$.

For $\omega_0^{FRM} + \overline{\alpha}_1 \omega_1^{FRM} \in \Omega_T^2$,

$$\frac{\partial \mathbf{E}[\pi_T(B_{FRM}(\alpha_1, \omega_1^{FRM}))]}{\partial c_T} = -\mathbf{1}'\mathbf{K}^{\mathbf{1}}_{\mathbf{T}}(1+a) > \frac{\partial \mathbf{E}[\pi_T(B_{-FRM}(\alpha_1))]}{\partial c_T}$$

and we obtain $\Upsilon^{c_T} > 0$. This concludes the proof for part (ii). ■

Name	Meaning
(ω_0, ω_1)	cash and asset holdings of the firm, called the firm's endowment
β	proportion of F_{FRM} deducted from cash holdings of the firm
α_0	stage 0 price of tradable asset
(F_T, c_T)	fixed and variable capacity costs of technology T
γ_T	salvage rate of fixed cost of technology T
F_{FRM}	fixed cost of financial risk management (FRM)
B	stage 1 budget
(a, E)	interest rate and credit limit of the loan contract
$r_f(= 0)$	risk-free rate
P	value of collateral physical asset
α_1	stage 1 price of tradable asset
$\boldsymbol{\xi} = (\xi_1, \xi_2)$	multiplicative demand intercept in product markets
Σ	covariance matrix of $\boldsymbol{\xi}$
ρ	coefficient of correlation in $\boldsymbol{\xi}$
σ	standard deviation of ξ_1 and ξ_2)
Γ_T	optimal stage 2 operating profits
Π_T	optimal stage 2 equity value
π_T	optimal expected (stage 1) equity value
Λ^{FRM}	expected (stage 0) equity value of better technology with FRM
Λ^{-FRM}	expected (stage 0) equity value of better technology without FRM
Π^*	optimal expected (stage 0) equity value
Δ_T	Value of financial risk management with technology T

Table 5.1: Summary of Notation

Technical Appendix II

Appendix A

Proof of Proposition 11: The proof follows from Proposition 1 of Boyabatlı and Toktay (2006a) by substituting $F_T = 0$, $\gamma_T = 0$ and $a = a_T$. ∎

Proof of Proposition 12: The proof follows from Proposition 2 of Boyabatlı and Toktay (2006a) by substituting $F_T = 0$, $\gamma_T = 0$ and $a = a_T$, $E = E_T$. The open-form expressions for $\mathbf{K}^*_\mathbf{T}(\tilde{B})$ is given by

$$\begin{aligned}
\mathbf{K}^{0}_{\mathbf{D}}{}' &= \left(\left(\frac{\overline{\xi}\left(1+\frac{1}{b}\right)}{c_D}\right)^{-b}, \left(\frac{\overline{\xi}\left(1+\frac{1}{b}\right)}{c_D}\right)^{-b}\right)\\
\overline{\mathbf{K}}_{\mathbf{D}}{}' &= \left(\frac{B}{2c_D}, \frac{B}{2c_D}\right)\\
\mathbf{K}^{1}_{\mathbf{D}}{}' &= \left(\left(\frac{\overline{\xi}\left(1+\frac{1}{b}\right)}{c_D(1+a_D)}\right)^{-b}, \left(\frac{\overline{\xi}\left(1+\frac{1}{b}\right)}{c_D(1+a_D)}\right)^{-b}\right)\\
\overline{\overline{\mathbf{K}}}_{\mathbf{D}}{}' &= \left(\frac{E_D+B}{2c_D}, \frac{E_D+B}{2c_D}\right)\\
K^0_F &= \left(\frac{M_F\left(1+\frac{1}{b}\right)}{c_F}\right)^{-b}\\
\overline{K}_F &= \frac{B}{c_F}\\
K^1_F &= \left(\frac{M_F\left(1+\frac{1}{b}\right)}{c_F(1+a_F)}\right)^{-b}\\
\overline{\overline{K}}_F &= \frac{E_F+B}{c_F}.
\end{aligned}$$

where $M_F = \mathbf{E}\left[\left(\xi_1^{-b} + \xi_2^{-b}\right)^{-\frac{1}{b}}\right]$. ∎

Proof of Corollary 7: The proof follows from Corollary 1 of Boyabatlı and Toktay (2006a) by substituting $F_T = 0$, $\gamma_T = 0$ and $a = a_T$, $E = E_T$. The expected (stage 1) equity value of the firm with a given budget level $\tilde{B}$ follows directly from Proposition 12:

$$\pi_T(\tilde{B}) = \begin{cases} \tilde{B} + \frac{c_T \mathbf{1}'\mathbf{K_T^0}}{-(b+1)} + P & \text{if } \tilde{B} \in \Omega_T^0 \\ M_T\left(\frac{\tilde{B}}{c_T}\right)^{1+\frac{1}{b}} + P & \text{if } \tilde{B} \in \Omega_T^1 \\ \tilde{B}(1+a_T) + \frac{c_T \mathbf{1}'\mathbf{K_T^1}(1+a_T)}{-(b+1)} + P & \text{if } \tilde{B} \in \Omega_T^2 \\ -E_T(1+a_T) + M_T\left(\frac{E_T+\tilde{B}}{c_T}\right)^{1+\frac{1}{b}} + P & \text{if } \tilde{B} \in \Omega_T^3 \end{cases} \quad (5.46)$$

where $M_F = \mathbf{E}\left[\left(\xi_1^{-b} + \xi_2^{-b}\right)^{-\frac{1}{b}}\right]$ and $M_D = \left(\overline{\xi}_1^{-b} + \overline{\xi}_2^{-b}\right)^{-\frac{1}{b}}$. ■

Proof of Proposition 13: The proof follows from Case (i) of Proposition 3 of Boyabatlı and Toktay (2006a) by substituting $F_T = 0$, $\gamma_T = 0$, $F_{FRM} = 0$ and $a = a_T$, $E = E_T$. ■

Proof of Proposition 14: The existence and the uniqueness of the variable cost threshold $\overline{c}_F(c_D, \mathbf{a}, \mathbf{H}^*)$, and the closed-form characterization of the symmetric case ($\overline{c}_F^S(c_D)$) follow from Proposition 4 of Boyabatlı and Toktay (2006a) by substituting $F_T = 0$, $\gamma_T = 0$, $F_{FRM} = 0$ and $a = a_T$, $E = E_T$. The dominance of investing in T^* over not making any technology investment follows from Case (i) of Proposition 11 of Boyabatlı and Toktay (2006a) by using $F_{FRM} = 0$. ■

Proof of Proposition 15: We only demonstrate the proof for the comparative static result with respect to a_T. We have

$$\frac{\partial EE_T}{\partial a_T} = -\overline{B} + c_T \mathbf{1}'\mathbf{K_T^0} \frac{1 + \frac{a_T b}{1+a_T}}{(1+a_T)^{-b}} \quad (5.47)$$

for $\overline{B} \in \Omega_T^2$. We define $x \doteq 1 + a_T$ and after some algebra to (5.47), we obtain $P(x) = -\frac{\overline{B}}{c_T \mathbf{1}'\mathbf{K_T^0}} x^{-b+1} + (b+1)x - b$. We have

$$\frac{\partial}{\partial x} P(x) = -(b+1)\left[-\frac{\overline{B}}{c_T \mathbf{1}'\mathbf{K_T^0}} x^{-b} - 1\right] < 0.$$

Since $P(1) > 0$ and $\lim_{x\to\infty} P(x) \to -\infty$, it follows that $\exists\ \underline{a}_T > 0$ (that solves $\frac{\partial}{\partial a_T} EE_T = 0$) such that $\frac{\partial}{\partial a_T} EE_T > 0$ for $a_T < \underline{a}_T$ and $\frac{\partial}{\partial a_T} EE_T < 0$ for $a_T > \underline{a}_T$. ■

Proof of Proposition 16: P_T is obtained from (3.6) after some algebra. We define $C_T \doteq O_T(1+\frac{1}{b})\left[1 - \frac{\overline{B}}{c_T \mathbf{1}'\mathbf{K_T^1}}\right]$ for technology $T \in \{D, F\}$. The proof follows by establishing $\frac{\partial}{\partial \overline{B}} C_T < 0$, $\frac{\partial}{\partial c_T} C_T < 0$ and $\frac{\partial}{\partial a_T} C_T < 0$. ■

Proof of Proposition 17: We will only consider the case $\overline{B} < c_T \mathbf{1}'\mathbf{K_T^0}$, otherwise the firm does not borrow from the creditor for any $a_T \geq 0$. Since the firm only borrows from the creditor if $\overline{B} < c_T \mathbf{1}'\mathbf{K}_\mathbf{T}^0(1+a_T)^b$ (as follows from (3.6)), for Pareto-optimality, it is sufficient to show that the expected (stage 0) equity value of the firm with technology T strictly decreases in the unit financing cost a_T. This follows from Case *ii* and Case *iii* of Lemma 6 in the proof of Proposition 4 in Boyabatlı and Toktay (2006a) by substituting $H = \omega_1$, $F_{FRM} = F_T = \gamma_T = 0$ and $a = a_T, E = E_T$ and using the identity $E_T = \frac{P}{1+a_T}$. Therefore the firm prefers the smallest a_T that satisfies $\mathbf{E}\left[\Lambda_T(a_T)\right] = U$.

If there exists a feasible $a_T^{'}$ ($\overline{B} < c_T \mathbf{1}'\mathbf{K}_\mathbf{T}^0(1+a_T^{'})^b$) such that $\mathbf{E}\left[\Lambda_T(a_T^{'})\right] \geq U$, since $\mathbf{E}\left[\Lambda_T(0)\right] < 0$ and $\mathbf{E}\left[\Lambda_T(a_T)\right]$ is a continuous function of a_T (which can be easily verified), it follows from the Mean-value theorem that such $a_T^* \leq a_T^{'}$ always exists and is unique. If there does not exist a feasible a_T that satisfies $\mathbf{E}\left[\Lambda_T(a_T)\right] = U$ then in equilibrium the creditor does not offer any contract. ■

Proof of Proposition 18: To prove the first part of the proposition, since $E_T^* = \frac{P}{1+a_T^*}$, it is sufficient to focus on a_T^*. We will only provide the proof for the results related to the bankruptcy cost BC. The results related to the underwriter fee U follow in a similar fashion. Let $a_T^0 < \infty$ and a_T^1 denote the equilibrium financing cost with bankruptcy cost BC^0 and BC^1 with $BC^1 > BC^0$, respectively. It follows from Proposition 17 that $a_T^0 = argmin_{a_T \geq 0}\ \mathbf{E}\left[\Lambda_T(a_T, BC^0)\right] = U$. Let us assume that the unit financing cost decreases in BC, i.e. $a_T^1 < a_T^0$. We will show by contradiction that this is not possible. Since $a_T^0 < \infty$, we also have $a_T^1 = argmin_{a_T \geq 0}\ \mathbf{E}\left[\Lambda_T(a_T, BC^1)\right] = U$. From $\frac{\partial}{\partial BC}\mathbf{E}\left[\Lambda_T(a_T, BC^0)\right] = -P_T < 0$, we obtain $U = \mathbf{E}\left[\Lambda_T(a_T^1, BC^1)\right] < \mathbf{E}\left[\Lambda_T(a_T^1, BC^0)\right]$. It follows from *i*)$\mathbf{E}\left[\Lambda_T(0, BC^0)\right] < 0$, *ii*) the continuity of $\mathbf{E}\left[\Lambda_T(a_T, BC^0)\right]$ in a_T, and *iii*) the Mean-value theorem that there exist $a_T^2 < a_T^1$ such that $\mathbf{E}\left[\Lambda_T(a_T^2, BC^0)\right] = U$. This is a contradiction with a_T^0 being the Pareto-optimal equilibrium for BC^0. Therefore, if there exists a_T^1 that satisfies $\mathbf{E}\left[\Lambda_T(a_T^1, BC^1)\right] = U$, we have $a_T^1 > a_T^0$. If such a_T^1 does not exist, then we have $a_T^1 = \infty > a_T^0$. In the same line of reasoning, it can be shown that if a feasible contract does not exist with BC^0, i.e. $a_T^0 = \infty$, we also have $a_T^1 = \infty$. This concludes the proof of the first part.

For the second part, the effect of increasing BC on the expected (stage 0) equity value of the firm follows from Case *ii* and Case *iii* of Lemma 6 in the proof of Proposition 4 in Boyabatlı and Toktay (2006a) by substituting $H = \omega_1$, $F_{FRM} = F_T = \gamma_T = 0$ and $a = a_T, E = E_T$ and using the identity $E_T = \frac{P}{1+a_T}$. The result related to the expected capacity investment level can be established in a similar fashion and is omitted. ■

Proof of Proposition 19: When $U = BC = 0$, it follows from (3.5) and Proposition 17 that $a_T^* = argmin_{a_T \geq 0}\, a_T\, \mathbf{E}[e_T] = 0$. This implies that $a_T^* = 0$ (minimum feasible unit financing cost) and from Proposition 17, we obtain $E_T^* = P$. ■

Proof of Proposition 20: If the capital markets are perfect, Proposition 19 states that $a_T^* = 0$ and $E_T^* = P$. Since $P > c_T \mathbf{1}'\mathbf{K_T^0}$ by assumption, with this financing cost scheme we obtain from Proposition 12 that $\Omega_T^1 = \Omega_T^3 = \emptyset$, $\mathbf{K_T^1} = \mathbf{K_T^0}$, and for $\tilde{B} \in \Omega_T^2$, the expected (stage 1) equity value is $\tilde{B} + \frac{c_T \mathbf{1}'\mathbf{K_T^0}}{-(b+1)} + P$. Therefore, we have from Proposition 12 that the firm invests in the budget-unconstrained capacity investment level for any budget realization $\mathbf{K_T^*}(\tilde{B}) = \mathbf{K_T^0}$, and borrows to finance this capacity level $e_T^*(\tilde{B}) = [c_T \mathbf{1}'\mathbf{K_T^0} + -B]^+$. It follows that expected (stage 1) equity value at each budget state $\tilde{B} > 0$ is $\tilde{B} + \frac{c_T \mathbf{1}'\mathbf{K_T^0}}{-(b+1)} + P$, and the expected (stage 0) equity value is $\overline{B} + \frac{c_T \mathbf{1}'\mathbf{K_T^0}}{-(b+1)} + P$. From Corollary 2 of Boyabatlı and Toktay (2006a), financial risk management does not have any value. With $a_D^* = a_F^* = 0$, and $E_D^* = E_F^* = P$, Proposition 14 implies that the technology choice T^* is determined by the variable cost threshold $\bar{c}_F^P(c_D) = \bar{c}_F^S(c_D)$ and T^* is more profitable than not investing in technology option. ■

Proof of Proposition 21: With uniform $[0, 2\bar{\xi}]$ distribution of ξ, the expected returns of the creditor can be written as

$$\mathbf{E}[\Lambda(a)] = \left(cK^0(1+a)^b - \overline{B}\right)\left(a - \frac{BC\,\bar{\xi}(1+1/b)}{2\bar{\xi}cK^0(1+a)^b}\right) \tag{5.48}$$

for a given a satisfying $\overline{B} \in \Omega_2(a)$. We first focus on the second term, the expected unit marginal profit of lending. We define $x \doteq 1 + a$ and after some algebra to the second term in (5.48), we obtain $G(x) = -\frac{BC\,\bar{\xi}(1+1/b)}{2\bar{\xi}cK^0}x^{-b} + x - 1$. We have $\frac{\partial}{\partial x}G(x) > 0$ for $x < \bar{x} = \left(\frac{2\bar{\xi}cK^0}{-bBC\,\bar{\xi}(1+1/b)}\right)^{\frac{-1}{b+1}}$ and $\frac{\partial}{\partial x}G(x) < 0$ for $x > \bar{x}$. Note that if $G(\bar{x}) < 0$ then the marginal profit is always negative and the creditor does not lend in equilibrium. This is the case if $BC > \widehat{BC} = \left(\frac{\bar{\xi}(1+1/b)^2}{c}\right)^{-b-1}\frac{2\bar{\xi}}{-b}$. We now show that for $BC \leq \widehat{BC}$ and for sufficiently small U and $\overline{B}$, the creditor always offers a contract and the firm borrows in equilibrium.

We obtain $\lim_{x\to\infty} P(x) = -\infty$ and $\frac{\partial}{\partial x}G(x)\big|_{x=1} > 0$ for $BC \leq \widehat{BC}$; therefore there exist two positive roots x_1, x_2 such that $1 < x_1 < \bar{x} < x_2$. Since we focus on Pareto-optimal equilibrium, we are interested in the smallest root x_1. Since the feasible set of the unit financing cost a is $[0, \left(\frac{cK^0}{\overline{B}}\right)^{-1/b} - 1]$, we are interested in the roots of $G(x) = 0$ in the range of $1 \leq x \leq \bar{\bar{x}}$ where $\bar{\bar{x}} = \left(\frac{cK^0}{\overline{B}}\right)^{-1/b}$. We now check if $a = x_1 - 1$ is feasible. For x_1 to be infeasible, i.e. $\bar{\bar{x}} < x_1$, the conditions

$$G(\bar{\bar{x}}) = -\frac{BC\,\bar{\xi}(1+1/b)}{2\bar{\xi}cK^0}\left(\frac{cK^0}{\overline{B}}\right) + \left(\frac{cK^0}{\overline{B}}\right)^{-1/b} - 1 < 0,$$

$$\left.\frac{\partial G(x)}{\partial x}\right|_{x=\bar{\bar{x}}} = \frac{b\,BC\,\bar{\xi}(1+1/b)}{2\bar{\xi}cK^0}\left(\frac{cK^0}{\overline{B}}\right)^{\frac{b+1}{b}} + 1 > 0$$

should be satisfied. The second condition is equivalent to $\overline{B} > \widehat{B} = c\left(\frac{-b\,BC}{2\xi}\right)^{\frac{b}{b+1}}$. Two cases can arise:

Case $i, \overline{B} \leq \widehat{B}$: We have $\frac{\partial}{\partial x}G(x)\big|_{x=\bar{\bar{x}}} < 0$, therefore x_1 is feasible. Since $x_1 < \bar{\bar{x}}$, for $U = 0$, $a^* = x_1 - 1 < \left(\frac{cK^0}{\overline{B}}\right)^{-1/b} - 1$ is the equilibrium financing cost. For $U > 0$, if U is sufficiently small then there exists an equilibrium financing cost that satisfies $a^* > x_1 - 1$ and $a^* < \left(\frac{cK^0}{\overline{B}}\right)^{-1/b} - 1$. If U is large enough, it is not feasible to generate U because both terms in (5.48) are bounded.

Case $ii, \overline{B} > \widehat{B}$: We have $\frac{\partial}{\partial x}G(x)\big|_{x=\bar{\bar{x}}} > 0$. Using $G(\bar{\bar{x}})$, we define

$$H(\overline{B}) \doteq -\frac{BC\,\bar{\xi}(1+1/b)}{2\bar{\xi}cK^0}\left(\frac{cK^0}{\overline{B}}\right) + \left(\frac{cK^0}{\overline{B}}\right)^{-1/b} - 1.$$

It is easy to establish that $\frac{\partial}{\partial B}H(\overline{B}) > 0$ for $\overline{B} < \widehat{B}$ and $\frac{\partial}{\partial B}H(\overline{B}) < 0$ for $\overline{B} > \widehat{B}$.

For $BC = \widehat{BC}$, we obtain $H(\widehat{B}) = 0$ and it follows that for $BC = \widehat{BC}$, $H(\overline{B}) < 0$ for $\overline{B} > \widehat{B}$ and hence $G(\bar{\bar{x}}) < 0$ is satisfied for such $\overline{B}$. Therefore, x_1 is infeasible. In this case, the creditor offers $a^* = \left(\frac{cK^0}{\overline{B}}\right)^{-1/b} - 1$ and the firm does not borrow for $U = 0$, and the creditor does not offer a contract for $U > 0$.

For $BC < \widehat{BC}$, we have $H(\widehat{B}) = 0$, therefore for some $\overline{B} > \widehat{B}$, we have $G(\bar{\bar{x}}) > 0$, and x_1 is feasible. For such $\overline{B}$, the creditor offers $a^* = x_1 - 1 < \left(\frac{cK^0}{\overline{B}}\right)^{-1/b} - 1$ for $U = 0$, and $a^* > x_1 - 1$ and $a^* < \left(\frac{cK^0}{\overline{B}}\right)^{-1/b} - 1$ for sufficiently small $U > 0$. For significantly large U, the creditor does not offer any contract. Since $\frac{\partial}{\partial B}H(\bar{B}) < 0$ for $\overline{B} > \widehat{B}$, after sufficient increase in $\overline{B}$, we may have $G(\bar{\bar{x}}) < 0$, and x_1 becomes infeasible. In this case, the creditor offers $a^* = \left(\frac{cK^0}{\overline{B}}\right)^{-1/b} - 1$ and the firm does not borrow for $U = 0$, and the creditor does not offer a contract for $U > 0$.

In summary,

1. If $BC > \widehat{BC}$ then for $U = 0$, the creditor offers a $a^* = \left(\frac{cK^0}{\overline{B}}\right)^{-1/b} - 1$ and the firm does not borrow in equilibrium; and the creditor does not offer any contract for $U \geq 0$,

2. If $BC \leq \widehat{BC}$ then

   i. for $U = 0$, if $\overline{B}$ is sufficiently small then the creditor offers a $a^* < \left(\frac{cK^0}{\overline{B}}\right)^{-1/b} - 1$ and the firm borrows in equilibrium,

   ii. for $U > 0$, if $\overline{B}$ and U is sufficiently small then the creditor offers a $a^* < \left(\frac{cK^0}{\overline{B}}\right)^{-1/b} - 1$ and the firm borrows in equilibrium,

   iii. for $U > 0$, if $\overline{B}$ is sufficiently small and U is sufficiently large then the creditor does not offer any contract,

   iv. for $U = 0$, if $\overline{B}$ is sufficiently large then the creditor offers $a^* = \left(\frac{cK^0}{\overline{B}}\right)^{-1/b} - 1$ and the firm does not borrow in equilibrium,

   v. for $U > 0$, if $\overline{B}$ is sufficiently large then the creditor does not offer any contract.

■

Proof of Proposition 22: We prove this result for general uniform distributions with mean $\bar{\xi}$ and support $[\bar{\xi} - d, \bar{\xi} + d]$ where $d \leq \bar{\xi}$. We use the mean-preserving spread of the uniform distribution to characterize an increase in the product market variability. For uniform distributions, this can be achieved by symmetrically increasing the support by keeping the mean constant, i.e. $[\bar{\xi} - d - \epsilon, \bar{\xi} + d + \epsilon]$ for $\epsilon > 0$. Higher ϵ leads to a higher variance of ξ_1[2]. Similar to (5.48), we have

$$\mathbf{E}\left[\Lambda(a)\right] = \left(cK^0(1+a)^b - \overline{B}\right)a - BC\left[\frac{\bar{\xi}(1+1/b)\left(1 - \frac{\overline{B}}{c_D\mathbf{1}'\mathbf{K_D^1}}\right) - (\bar{\xi} - d - \epsilon)}{2(\bar{\xi} + d)}\right]$$

for a given a satisfying $\overline{B} \in \Omega_2(a)$. We obtain

$$\frac{\partial \mathbf{E}\left[\Lambda(a)\right]}{\partial \epsilon} = \frac{BC}{2(\bar{\xi} + d)^2}\left[\bar{\xi}(1+1/b)\left(1 - \frac{\overline{B}}{c_D\mathbf{1}'\mathbf{K_D^1}}\right) - \bar{\xi}\right] < 0$$

for any a. Since the expected return of the creditor decreases in ϵ, within similar arguments of Proposition 18, it follows that $\frac{\partial}{\partial \epsilon}a^* \geq 0$. The effect of increasing a^* on the expected (stage 0) equity value of the firm follows from Case *ii* and Case *iii* of Lemma 6 in the proof of Proposition 4 in Boyabatlı and Toktay (2006a) by substituting $H = \omega_1$, $F_{FRM} = F_D = \gamma_D = 0$ and using the identity $E = \frac{P}{1+a}$. The result related to the expected capacity investment level can be established in a similar fashion and is omitted. ■

[2]Variance is the correct indicator of risk for uniform distributions in the Rothschild-Stiglitz sense (Eeckhoudt and Gollier 1995, p.82)

Proof of Proposition 23: From (5.48), we obtain

$$\frac{\partial \mathbf{E}[\Lambda(a)]}{\partial \overline{B}} = -\left(a - \frac{BC\,\overline{\xi}(1+1/b)}{2\overline{\xi}cK^0(1+a)^b}\right) \leq 0$$

for any a satisfying $\overline{B} \in \Omega_2(a)$. Since the expected return of the creditor decreases in $\overline{B}$, within similar arguments of Proposition 18, it follows that $\frac{\partial}{\partial \overline{B}}a^* \geq 0$. From Case *ii* and Case *iii* of Lemma 6 in the proof of Proposition 4 in Boyabatlı and Toktay (2006a) by substituting $H = \omega_1$, $F_{FRM} = F_D = \gamma_D = 0$ and using the identity $E = \frac{P}{1+a}$, it follows that the expected (stage 0) equity value of the firm decreases in a. If the firm borrows in equilibrium, i.e. $\overline{B} \in \Omega_2(a^*)$, then the expected optimal capacity investment level $K^* = \left(\frac{\overline{\xi}(1+\frac{1}{b})}{c(1+a^*)}\right)^{-b}$ decreases in $\overline{B}$ because of increasing a^*. If the firm does not borrow in equilibrium, i.e. $\overline{B} \in \Omega_0 1$, then the expected optimal capacity investment level

$$K^* = \begin{cases} \left(\frac{\overline{\xi}(1+\frac{1}{b})}{c(1+a^*)}\right)^{-b} & \text{if } \overline{B} \in \Omega_F^0 \\ \frac{\overline{B}}{c} & \text{if } \overline{B} \in \Omega_F^1 \end{cases}$$

increases in $\overline{B}$. It is easy to establish that the expected (stage 0) equity value of the firm increases in $\overline{B}$ without considering the effect on the equilibrium level of financing cost. For the effect on the equity value, we have two drivers, $\overline{B}$ and a^* that work in opposite directions. Figure 5.2 demonstrates that either effect may dominate; therefore expected (stage 0) equity value of the firm may increase or decrease in the expected budget level $\overline{B}$. The numerical example is generated by using the parameter levels $c = 1, b = -2, P = 650, \overline{\xi} = 25, BC = 10, U = 10$. ■

Proof of Proposition 24: From (5.48), we obtain

$$\frac{\partial \mathbf{E}[\Lambda(a)]}{\partial c} = (b+1)K^0(1+a)^b\left(a - \frac{BC\,\overline{\xi}(1+1/b)}{2\overline{\xi}cK^0(1+a)^b}\right) + \left(cK^0(1+a)^b - \overline{B}\right)\frac{(b+1)BC\,\overline{\xi}(1+1/b)}{2\overline{\xi}c^2K^0(1+a)^b} < 0$$

for any a satisfying $\overline{B} \in \Omega_2(a)$. Since the expected return of the creditor decreases in ϵ, within similar arguments of Proposition 18, it follows that $\frac{\partial}{\partial c}a^* \geq 0$. The proof for the effect of increasing a^* on the expected optimal capacity investment and (stage 0) equity value of the firm is similar to Proposition 22 and is omitted. ■

Proof of Proposition 25: Without financial risk management, the expected return of the creditor is given by

$$\mathbf{E}[\Lambda(a)] = \left(a - \frac{BC\,\overline{\xi}(1+1/b)}{2\overline{\xi}cK^0(1+a)^b}\right)\int_{\omega_0}^{cK^0(1+a)^b}\left(cK^0(1+a)^b - \tilde{B}\right)dR_{B(0)}(\tilde{B}) \qquad (5.49)$$

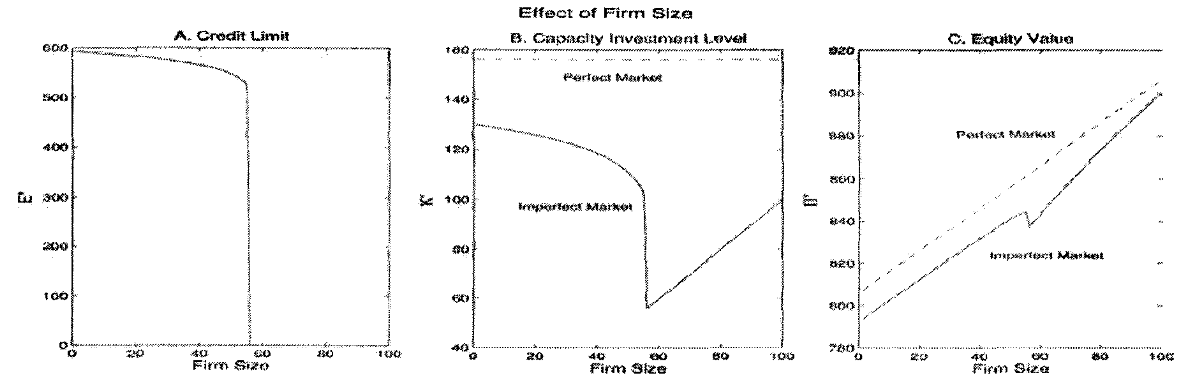

Figure 5.2: Increasing firm size (expected budget level) increases a^* and decreases E^* (Panel A). The creditor does not offer a loan contract after sufficient increase in $\overline{B}$. The capacity level decreases in $\overline{B}$ when the firm borrows and increases in $\overline{B}$ when the firm does not borrow (Panel B). For small levels of $\overline{B}$, the positive effect of an increase in $\overline{B}$ dominates the negative effect of an increase in a^* and the equity value increases in $\overline{B}$. With a sharp increase in a^*, the equity value decreases (Panel C).

where $R_{B(0)}(\tilde{B}) = R_{\alpha_1}\left(\frac{\tilde{B}-\omega_0}{\omega_1}\right)$ for $\tilde{B} \geq \omega_0$ as follows from the proof of Proposition 3 of Boyabatlı and Toktay (2006a). We analyze the equilibrium financing cost a^*_{-FRM} by focusing on each case of a^*_{FRM} equilibrium that we demonstrated in Proposition 21 separately.

1. For $BC > \widehat{BC}$ and $U = 0$, we have $a^*_{FRM} = \left(\frac{cK^0}{\overline{B}}\right)^{-1/b} - 1$. Since the first term in (5.49), the expected marginal profit, is identical with financial risk management case; it follows that $a^*_{-FRM} = \left(\frac{cK^0}{\omega_0}\right)^{-1/b} - 1$ and the firm does not borrow in equilibrium. Therefore there is no strategic value of financial risk management. For $U > 0$, similar to Proposition 21, the creditor does not offer any contract and there is no strategic value of financial risk management.

2. If $BC \leq \widehat{BC}$ then

   i. for $U = 0$ and sufficiently small $\overline{B}$, a^*_{FRM} is the financing cost that makes the marginal profit term 0 ($a^*_{FRM} + 1$ is a solution to $G(x)$ as defined in Proposition 21). Since $\left(\frac{cK^0}{\omega_0}\right)^{-1/b} - 1 > \left(\frac{cK^0}{\overline{B}}\right)^{-1/b} - 1$ and the expected marginal profit term is independent of budget level, we have $a^*_{FRM} = a^*_{-FRM}$, the firm borrows in equilibrium and there is no strategic value of financial risk management.

ii. for $U > 0$, if U and $\overline{B}$ are sufficiently small such that a finite a^*_{FRM} exists then we have $a^*_{-FRM} < a^*_{FRM}$ and there is negative strategic value of financial risk management. To prove this result, we define

$$f(\tilde{B}) \doteq \begin{cases} c_T \mathbf{1}'\mathbf{K}^{\mathbf{1}}_{\mathbf{T}} - \tilde{B} & \text{if } \tilde{B} \leq c_T \mathbf{1}'\mathbf{K}^{\mathbf{1}}_{\mathbf{T}} \\ 0 & \text{if } \tilde{B} > c_T \mathbf{1}'\mathbf{K}^{\mathbf{1}}_{\mathbf{T}} \end{cases}$$

Since $f(\tilde{B})$ is a convex function, we have $\mathbf{E}[f(\tilde{B})] \geq f(\overline{B})$ therefore the expected amount of lending with financial risk management is lower than without financial risk management. This induces the creditor to charge $a^*_{-FRM} < a^*_{FRM}$ in equilibrium. Since the expected (stage 0) equity value is decreasing in a, it follows that $\Pi^{FRM}(a^*_{FRM}) < \Pi^{FRM}(a^*_{-FRM})$ and there is negative strategic value.

iii. for $U > 0$, if $\overline{B}$ is sufficiently small and U is sufficiently large such that a finite a^*_{FRM} does not exist then three cases may happen. If U is so large, even if expected lending is higher without financial risk management and the creditor needs to charge lower a to attain U, such an a may not be feasible and the creditor does not lend in equilibrium without financial risk management either, i.e. $a^*_{FRM} = a^*_{-FRM} = \infty$. In this case, there is no strategic value of financial risk management. Since expected lending amount is higher without financial risk management, a finite a^*_{-FRM} may exist. Since $a^*_{FRM}\infty$ and the firm does not borrow, such a^*_{FRM} has the same effect with $a^*_{FRM} = \left(\frac{cK^0}{\overline{B}}\right)^{-1/b} - 1$ because the firm does not borrow at this financing cost either. If $a^*_{-FRM} > \left(\frac{cK^0}{\overline{B}}\right)^{-1/b} - 1$, then we have $\Pi^{FRM}(a^*_{FRM}) - \Pi^{FRM}(a^*_{-FRM}) = 0$ because the firm does not borrow at each financing cost with financial risk management. Therefore, there is no strategic value. If the increase in expected lending is sufficiently high, then we may have $a^*_{-FRM} < \left(\frac{cK^0}{\overline{B}}\right)^{-1/b} - 1$. In this case, the firm borrows at a^*_{-FRM} with financial risk management, and since $a^*_{-FRM} < a^*_{FRM}$ we have negative strategic value of financial risk management.

iv. for $U = 0$, if $\overline{B}$ is sufficiently large such that there is no feasible a that makes the marginal profit term with financial risk management equal to 0, two cases may happen. Recall from Proposition 21) that $\overline{B} > \widehat{B} = c\left(\frac{-b\,BC}{2\xi}\right)^{\frac{b}{b+1}}$ should hold for this case to be satisfied. We have $\overline{B} = \omega_0 + \overline{\alpha}_1\omega_1$. If ω_0 is sufficiently large, an equivalent condition $\omega_0 > c\left(\frac{-b\,BC}{2\bar{\xi}}\right)^{\frac{b}{b+1}}$ can be satisfied without financial risk management. If ω_0 also satisfies the second necessary condition for infeasibility, $-\frac{BC\,\bar{\xi}(1+1/b)}{2\bar{\xi}cK^0}\left(\frac{cK^0}{\omega_0}\right)+\left(\frac{cK^0}{\omega_0}\right)^{-1/b}-1<0$ then there is no feasible a that makes the marginal profit term without financial risk management equal to 0. In this case we obtain $a^*_{-FRM} = \left(\frac{cK^0}{\omega_0}\right)^{-1/b}-1$, the firm does not borrow in equilibrium and there is no strategic value of financial risk management. If ω_0 is not very large, then the infeasibility conditions cannot be satisfied. In this case there exists a $a^*_{-FRM} < \left(\frac{cK^0}{\omega_0}\right)^{-1/b}-1$. We have $a^*_{FRM} < a^*_{-FRM}$ otherwise there should be a $a^*_{FRM} < \left(\frac{cK^0}{\overline{B}}\right)^{-1/b}-1$ that makes the marginal profit term with financial risk management equal to 0. Similar to Case *iii*, there is no strategic value of financial risk management.

iv. for $U > 0$, if $\overline{B}$ is sufficiently large such that there is no feasible a that makes the marginal profit term with financial risk management equal to 0, without financial risk management either there is a finite $a^*_{-FRM} < \left(\frac{cK^0}{\omega_0}\right)^{-1/b}-1$ or the creditor does not offer a contract. The proof is similar to Case *iv* and is omitted. In either case, there is no strategic value of financial risk management.

In the proposition, 1 corresponds to Cases *i* and *iv*, 2 corresponds to Case *ii*, 3 to Case *iii* and 4 to Case *v*. This concludes the proof. ■

Proof of Proposition 26: It follows from (5.46) that the expected (stage 0) equity value of the firm is independent of the covariance matrix $\mathbf{\Sigma}$ for a given financing cost scheme (a_T, E_T). From Propositions 15 and 16, we obtain

$$\mathbf{E}\left[\Lambda_D(a_D)\right] = (c_D\mathbf{1}'\mathbf{K}^{\mathbf{1}}_{\mathbf{D}} - \overline{B})a_D - BC\;Pr\left(\xi_1+\xi_2 < 2\bar{\xi}(1+\frac{1}{b})\left[1-\frac{\overline{B}}{c_D\mathbf{1}'\mathbf{K}^{\mathbf{1}}_{\mathbf{D}}}\right]\right)$$

where $\mathbf{K}^{\mathbf{1}}_{\mathbf{D}}{}' = \left(\left(\frac{\bar{\xi}(1+\frac{1}{b})}{c_D(1+a_D)}\right)^{-b},\left(\frac{\bar{\xi}(1+\frac{1}{b})}{c_D(1+a_D)}\right)^{-b}\right)$. The first term (EE_D) is also independent of $\mathbf{\Sigma}$ and the effect of the demand correlation ρ and the demand variability σ is inherent in the expected default cost ED_D. If (ξ_1, ξ_2) is bivariate normal with $N(\boldsymbol{\xi}, \mathbf{\Sigma})$, $\xi_1+\xi_2$ is also Normally distributed with mean $2\bar{\xi}$ and standard deviation $\overline{\sigma} = \sigma\sqrt{2(1+\rho)}$. Recall from the proof of Proposition 16, C_D denotes the righthand side of the default probability P_D. Since $b < -1$ and $\overline{B} < c_D\mathbf{1}'\mathbf{K}^{\mathbf{1}}_{\mathbf{D}}$, we obtain $C_D < 2\bar{\xi}$. We have $P_D = Pr\left(\xi_1+\xi_2 < C_D\right) = \Phi\left(\frac{C_D-2\bar{\xi}}{\overline{\sigma}}\right)$ where $\Phi(.)$ is the cumulative distribution function of the standard normal random variable. For an arbitrary a_D that satisfies $\overline{B} \in \Omega^2_D(a_D)$, since

$\frac{\partial}{\partial\rho}\overline{\sigma} = \frac{\sigma}{\overline{\sigma}} > 0$, $\frac{\partial}{\partial\sigma}\overline{\sigma} = \sqrt{2(1+\rho)} > 0$, and $C_D < 2\overline{\xi}$ it follows that

$$\begin{aligned}\frac{\partial \mathbf{E}\left[\Lambda_D(a_D)\right]}{\rho} &= -BC\phi\left(\frac{C_D - 2\overline{\xi}}{\overline{\sigma}}\right)\left(\frac{2\overline{\xi} - C_D}{\overline{\sigma}^2}\right)\frac{\partial\overline{\sigma}}{\partial\rho} < 0,\\ \frac{\partial \mathbf{E}\left[\Lambda_D(a_D)\right]}{\sigma} &= -BC\phi\left(\frac{C_D - 2\overline{\xi}}{\overline{\sigma}}\right)\left(\frac{2\overline{\xi} - C_D}{\overline{\sigma}^2}\right)\frac{\partial\overline{\sigma}}{\partial\sigma} < 0\end{aligned}$$

where $\phi(.)$ is the density function of the standard normal random variable. Since the expected return of the creditor decreases in ρ and σ, within similar arguments of Proposition 18, it follows that $\frac{\partial}{\partial\rho}a_D^* \geq 0$ and $\frac{\partial}{\partial\sigma}a_D^* \geq 0$. The effect of increasing a_D^* on the expected (stage 0) equity value of the firm follows from Case *ii* and Case *iii* of Lemma 6 in the proof of Proposition 4 in Boyabatlı and Toktay (2006a) by substituting $H = \omega_1$, $F_{FRM} = F_D = \gamma_D = 0$ and $a = a_D, E = E_D$ and using the identity $E_D = \frac{P}{1+a_D}$. The result related to the expected capacity investment level can be established in a similar fashion and is omitted. ■

Proof of Proposition 27: Since $\boldsymbol{\xi}$ has a symmetric bivariate distribution, with perfect position correlation ($\rho = 1$), we have $Pr(\xi_1 = \xi_2) = 1$. In this case, we obtain from Proposition 16 that $H_F(\boldsymbol{\xi}) = H_D(\boldsymbol{\xi})$ for any $\boldsymbol{\xi}$ realization and $M_F = M_D$. With the identical financing cost scheme $a_F = a_D$ and $E_F = E_D$, we have $\overline{c}_F^S(c_D) = c_D$ from Proposition 14 and $c_D\mathbf{1}'\mathbf{K}_\mathbf{D}^\mathbf{1} = c_F K_F^1$ for $c_F = \overline{c}_F^S(c_D)$ from the proof of Lemma 7 of Boyabatlı and Toktay (2006a). Therefore, we have $P_F = P_D$ and $EE_D = EE_F$ for an arbitrary $a_D = a_F$. Since the expected returns of the creditor is identical with both technologies for a given unit financing cost, the creditor offers the same unit financing cost for both technologies ($a_D^*(\mathbf{K}_\mathbf{D}^*) = a_F^*(K_F^*)$) in equilibrium. From Lemma 7 of Boyabatlı and Toktay (2006a), we conclude that the firm's optimal total capacity investment decision $\mathbf{1}'\mathbf{K}_\mathbf{T}^*$, and the expected (stage 0) equity value of the firm $\pi_T(\mathbf{K}_\mathbf{T}^*; a_T^*(\mathbf{K}_\mathbf{T}^*))$ is identical for each technology. ■

Proof of Proposition 28: With the identical financing cost scheme $a_F = a_D$ and $E_F = E_D$, for $c_F = \overline{c}_F^S(c_D)$, the firm is indifferent between two technologies (which follows from Proposition 14) and we have $c_D\mathbf{1}'\mathbf{K}_\mathbf{D}^\mathbf{1} = c_F K_F^1$ from the proof of Lemma 7 of Boyabatlı and Toktay (2006a). Therefore, with identical financing cost scheme, the expectation earning with each technology is the same ($EE_D = EE_F$) for the creditor. The default risk P_T comparison at the identical financing cost scheme (off-the-equilibrium path) between two technologies determines the ordering between the equilibrium level of unit financing costs, which in turn determines the technology choice of the firm in equilibrium. Figure 5.3 demonstrates the default region with each technology with identical financing cost scheme

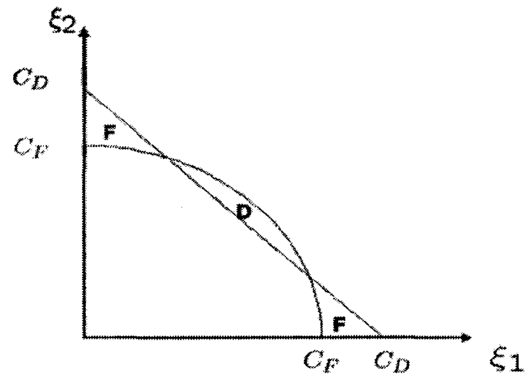

Figure 5.3: Default regions in (ξ_1, ξ_2) space with each technology: C_D and C_F are the right-hand side terms in the default probability as defined in the proof of Proposition 16. The area below the straight line (curve) is the default region with the dedicated (flexible) technology. F (D) represents the $\boldsymbol{\xi}$ realizations that the firm does not default with the flexible (dedicated) technology and defaults with the other technology.

for $c_F = \bar{c}_F^S(c_D)$. The overall default probability P_T is determined by superimposing the $\boldsymbol{\xi}$ distribution and taking the expectation over the regions. For the technology cost pair $\left(c_F^S(c_D;\rho), c_D\right)$, since $\left(\xi_1^{-b} + \xi_2^{-b}\right)^{-\frac{1}{b}} < \left((\xi_1+\xi_2)^{-b}\right)^{-\frac{1}{b}}$ for any $\boldsymbol{\xi}$ realization, it follows that $M_F < 2\bar{\xi}$ and we have $C_D = C_F\frac{2\bar{\xi}}{M_F} > 1$. Therefore the region denoted with F always exists around the points $(0, C_D)$ and $(C_D, 0)$. The point $(\frac{C_D}{2}, \frac{C_D}{2})$ that is on the default line of the dedicated technology is in the default region of the flexible technology if $\left(\frac{C_D}{2}^{-b} + \frac{C_D}{2}^{-b}\right)^{-\frac{1}{b}} \leq C_F$. For the technology cost pair $\left(c_F^S(c_D;\rho), c_D\right)$ this condition is equivalent to $2^{-\frac{1}{b}}\bar{\xi} \leq M_F$. It follows from Proposition 4 of Boyabatlı and Toktay (2006a), this condition is satisfied with equality only if $\rho = 1$, otherwise the inequality is always satisfied for $\rho \neq 1$. Therefore, D region only exists if $\rho \neq 1$. With close to perfect positive correlation, all the $\boldsymbol{\xi}$ realizations are located around $\xi_1 = \xi_2$ line (which also passes through the point $(\frac{C_D}{2}, \frac{C_D}{2})$). It follows that after taking the expectation over the default regions, we obtain $P_D < P_F$ because of the existence of region D. Since we have $P_D < P_F$ for identical financing cost scheme, the creditor charges lower financing cost for the dedicated technology in equilibrium $a_D^* < a_F^*$. It follows from Case *ii* and Case *iii* of Lemma 6 in the proof of Proposition 4 in Boyabatlı and Toktay (2006a) by substituting $H = \omega_1$, $F_{FRM} = F_T = \gamma_T = 0$ and $a = a_T, E = E_T$ and using the identity $E_T = \frac{P}{1+a_T}$ that the expected (stage 0) equity value of the firm decreases in a_T; therefore the firm chooses the dedicated technology in equilibrium. ∎